

AMCS 602 Fall 2017  
Homework Set IV, Due Oct. 18, 2017

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**Reading:** Read the Lecture 11 - 19 of Trefethen and Bau: *Numerical Linear Algebra*, which is published by SIAM. Page numbers below refer to this book. The solutions of the following problems should be carefully written up and handed in.

1. Page 85, Problem 11.1. (Hint: Use SVD decomposition.)

2. Assume that the polynomial  $p(x) = \sum_{k=0}^n a_k x^k$  with real coefficients has  $n$  distinct real roots. Show that if the  $i$ th coefficient  $a_i$  is perturbed by an infinitesimal quantity  $\delta a_i$ , then the perturbation of the  $j$ th root  $x_j$  is

$$\delta x_j = \frac{(\delta a_i) x_j^i}{p'(x_j)}.$$

3. Suppose that  $\{\epsilon_1, \dots, \epsilon_m\}$  are real numbers that all satisfy  $|\epsilon_i| \leq \eta$ . Show that given  $C > 1$ , we have that

$$\prod_{j=1}^m (1 + \epsilon_j) = 1 + \epsilon,$$

where  $|\epsilon| \leq Cm\eta$ , provided that  $0 < \eta \leq \min\{\frac{\log C}{m-1}, 1\}$ . For double precision arithmetic how large can we take  $m$  if  $C = 2$ ?

4. Page 96, problem 12.3.

5. Page 127, problem 17.2.

6. Page 143, problem 19.1.