Fun Short Puzzles

Compiled by Yao-Rui and Alon

I will try to give references to puzzles with known original source.

Puzzle 1 (Manhole Covers)**.** Why are manhole covers circular?

Puzzle 2 (Metal Ring)**.** If you heat a metal ring, will the inner circle increase or decrease in radius?

Puzzle 3 (Secret Numbers)**.** Suppose I have a secret list of 100 natural numbers. If you give me any list of 100 natural numbers, I will return you the dot product of my list with your list. Can you figure out my list by doing this twice?

Puzzle 4 (Jumping Frogs)**.** Suppose there are 101 lilypads arranged in a line, and 100 frogs occupying all but the middle lilypad. The frogs on the left can only move right, and the frogs on the right can only move left. In addition, a frog can jump over an adjacent frog to land on an empty lilypad. Find a strategy that gives the minimum number of jumps.

Puzzle 5 (Crystal Balls)**.** A given brand of crystal ball breaks if you drop it from a certain floor of a 100 story building. Find the minimal amount of tries needed to figure out what this floor is, if you are given only 2 crystal balls to experiment with. You are allowed to break both crystal balls to figure this out.

Puzzle 6 (Coin Problem)**.** Suppose you have an infinite amount of coins of only two different denominations. What is the maximum amount that cannot be obtained using these two types of coins?

Puzzle 7 (Coin Weighing)**.** Suppose I have 10 coins that looks identical. However, two coins are heavier than the others, and these two coins have the same weight (the other eight coins also have the same lighter weight). What is the minimum number of weighs you can perform on a perfect weighing scale to identify the two heavier coins?

Puzzle 8 (Flipping Cups)**.** Consider a rotating table shaped like a square. Your friend blindfolds you, and arranges four cups on the vertices of the table. Each cup is either faced up or faced down. At every turn, you are allowed to choose and feel two cups with both of your hands, flipping zero, one or both of them if desired. Your friend then rotates the table, permuting the vertices cyclically and randomly. Is there a strategy to make sure all four cups are all faced down or faced up after finitely many turns? What if an n-gon is considered instead of a square?

Puzzle 9 (Circular Table)**.** Suppose you have a circular table. You randomly make a leg for the table that is triangular prism shaped, and such that the vertices of the triangle are on the circumference of the table. What is the probability the table does not tip over? Can you generalize this to higher dimensions?

Puzzle 10 (Dice Throws)**.** What is the expected number of throws needed to get n number of a face of a dice? What is the expected number of throws needed to get all six faces of a dice?

Puzzle 11 (Walking on Earth)**.** Assume the Earth is a perfect sphere. Identify the points on Earth that allow the following to happen: choose a direction you wish and walk one mile; turn right 90-degrees and walk one mile; turn right 90-degrees and walk one mile; you end up in your starting position.

Puzzle 12 (Shape of Drums)**.** Must two drums of different shapes sound different? [*Source: Lipman Bers.*]

Puzzle 13 (Connecting Dots)**.** Without lifting your pen, demonstrate why the minimum number of lines to connect the four vertices of a square is 3. What is the minimum number of lines to connect the eight vertices of a cube without lifting your wand in three-dimensional space?

Puzzle 14 (Cake Frosting)**.** Suppose you have a spherical cake with evenly coated frosting. Arrange two knives parallel to each another and of fixed distance. Show that, no matter where you cut the cake, the amount of frosting you get in between the knives will always be the same.

Puzzle 15 (Brachistochrone)**.** Find the shape of the curve such that, with only gravity and no friction, a bead will slide along this curve from one point to another in the shortest time. [*Source: Johann Bernoulli.*]

Puzzle 16 (Hyperplane Arrangements)**.** Consider the space $\mathbb{R}^n$, together with an arrangement of finite hyperplanes. Can you count how many regions the hyperplanes cut the space into?

Puzzle 17 (Donut Cutting)**.** What is the maximum number of (possibly nonequal) pieces you can cut a regular donut into with five straight cuts?

Puzzle 18 (Coloring the Map)**.** Consider the regions of the world map separated by country borders (seas also count as regions). What is the minimum number of colors needed to color the regions such that no two adjacent regions have the same color? [*Source: Four Color Theorem.*]

Puzzle 19 (Coloring the Cube)**.** How many ways are there to color the six faces of the cube using 10 different colors?

Puzzle 20 (Triangulating Polygons)**.** How many ways can you pair the vertices of a regular $2n$-gon such that the line segments joining paired vertices do not intersect?

Puzzle 21 (Cutting Polytopes)**.** Can we cut up a polygon into finitely many pieces to form another polygon? How about polyhedra?

Puzzle 22 (Hanging Pictures)**.** Hang a picture on five nails such that removing any two nails will make the picture fall, but removing just one nail will not. Assume you have enough rope to hang the picture.

Puzzle 23 (Hairy Ball)**.** Can you comb a hairy (closed orientable) genus g surface without creating a cowlick? [*Source: Hairy Ball Theorem.*]

Puzzle 24 (Origami)**.** How do you trisect an angle via origami? It is known that this cannot be done with just a straightedge and a compass. (Similarly, doubling the cube can be done via origami, but not with a straightedge and compass.) [*Source: Rigid Origami.*]

Puzzle 25 (Making Surfaces)**.** Given a regular paper shaped like a $2n$-gon, try to glue sides together pairwise to form a (closed orientable) genus g surface. How many ways are there to do this? Note that there are two ways to glue a pair of sides. [*Source: Harer-Zagier Formula.*]

Puzzle 26 (Charged Particles)**.** Suppose we have 100 identical charged particles on a line in the complex plane, with the first and last particle held fixed. What will be the shape of these particles at strict stable equilibrium (i.e. minimizing the total energy)?

Puzzle 27 (Brick Collision)**.** Suppose we have a brick of mass 1. Consider the following one-dimensional frictionless setup. To the left of the brick is an unbreakable wall, and to the right of the brick is another brick of mass $10^{2(d-1)}$ for some positive integer d. Push this heavier brick to the left and track the number N of collisions between the two bricks (do not consider collisions between the lighter brick and the wall). Explain why the digit of N corresponds to the first d digits of π. For example, if $d = 5$, then $N = 31415$.

Puzzle 28 (Multizeta Values)**.** It is well-known that the sum of reciprocals of squares equals $\pi^2/6$. Now, the Riemann 2-multizeta value is defined to be

$$\zeta(2, \dots, 2) = \sum_{n_1 > n_2 > \cdots > n_k > 0} \frac{1}{n_1^2 \cdots n_k^2}.$$

Can you evaluate this sum?

Puzzle 29 (Almost Integers)**.** Why are $e^{\pi\sqrt{67}}$ and $e^{\pi\sqrt{163}}$ almost integers? [*Source: Srinivasa Ramanujan.*]

Puzzle 30 (Trimming a Tree)**.** Consider a rooted tree that is magical in the following sense. Suppose you cut a leaf connected to a vertex x. Suppose y is the parent of x. Then y will grow 100 new tree children T, where T is a copy of the subtree rooted at x with one less leaf. Is it possible to completely trim this tree such that only the root remains?

Puzzle 31 (Four Points Two Distances)**.** Find all configurations of four distinct points on the plane such that the set of pairwise distances has cardinality at most two.

Puzzle 32 (Dominoes and Chessboards)**.** Delete a block from a chessboard. Can you tile the chessboard with dominoes? How about deleting two blocks?

Puzzle 33 (Rooks and Chessboards)**.** Can you place 8 nonattacking rooks on a chessboard? How many ways are there?

Puzzle 34 (Queens and Chessboards)**.** Can you place 8 nonattacking queens on a chessboard? How many ways are there?

Puzzle 35 (Prisoners and Hats)**.** One hundred prisoners are arranged in a line facing forward, so each prisoner can only see the prisoners in front of them. Each prisoner is wearing a hat that is either blue or red. Starting from the back, every prisoner must guess their own hat color correctly, or be killed by the warden. What is the most optimal strategy for this, given that each prisoner can hear the guess of every other prisoner, and the prisoners are allowed to discuss a strategy beforehand?

Puzzle 36 (Prisoners and Lightbulbs)**.** A warden has thought up a game for 100 prisoners. He goes into an empty room and arranges one hundred light bulbs in a row, switched off. At each step, a single prisoner enters the room. If this prisoner is the k^{th} one to enter, he/she will flip the switch of all light bulbs at positions that are multiples of k. How many light bulbs will be switched on at the end of the procedure?

Puzzle 37 (Prisoners and Boxes)**.** The warden has thought up another game for 100 prisoners. He goes into an empty room and makes up 100 identical boxes in a row, writing each of the 100 prisoners' (unique) names in each box. Each prisoner is, once at a time, allowed to enter the room and open 50 boxes. After each prisoner's turn, the boxes are all closed and the next prisoner comes in, until the last one finishes his turn. If every prisoner opens the box that has his/her name in it, all are free to go. Else all prisoners gets killed. What is the most optimal strategy for this, given that the prisoners are allowed to discuss a strategy beforehand, and no communication is allowed after the game starts?

Puzzle 38 (Angels and Devils)**.** Fix a natural number k, and consider an infinite chessboard with an angel at the origin. On each turn, the angel can move at most k moves of a chess king. After that, the devail can remove a square not occupied by the angel. Will the devil be able to confine the angel? The angel can jump over removed squares, but not land on them. [*Source: John Horton Conway.*]

Puzzle 39 (Parking a Car)**.** Suppose there are n parking lots arranged in a line, and n cars lined up. Each car has its own preferred parking lot (and may not be unique). When it is the k^{th} car's turn to park, it will drive to its preferred parking lot; if it's taken, the car will park at the next available parking lot, and drive away if none exists. How many such preferences are there with the property that every car will be able to park at a parking lot? For example, if $n = 3$, then 213 and 111 are such preferences, but not 313 nor 222.

Meta-Puzzle. Read up some combinatorial games and their strategies, such as Nim, Sprout, Animal Chess, 3D Tic-Tac-Toe, and the Game of Life. Also try to find fun results like the Futurama Theorem.