

Math 114. Fall 2014. HW 7. Due Nov 26 Wednesday

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**Instructions for written homework.**

- You are encouraged to work with others on these problems. You are expected to write the solutions yourself.
  - Your solutions should be legible and well organized. **Graders will deduct points for solutions that are difficult to read, or are disorganized.** For the benefit of the grader, please turn in solutions to problems in the assigned order, i.e. #1, then #2, then #3, etc.
  - Staple your pages together. Do not turn in notebook paper with tattered edges. **Homework that is unstapled or is lacking a name will not be graded.**
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**Problem 1** (Fall 2010). Find the  $y$ -coordinate of the center of mass of a thin plate in the shape of the upper half of the unit circle:

$$x^2 + y^2 = 1; \quad y \geq 0$$

if the density  $\delta$  at the point  $(x, y)$  is  $\delta(x, y) = x^2 + y^2$ .

**Problem 2** (Fall 2012). A solid cylinder of height 1 and radius 1 is formed by all the points  $(x, y, z)$  such that  $x^2 + y^2 \leq 1$  and  $0 \leq z \leq 1$ . Find the center of mass of the cylinder if its density at  $(x, y, z)$  is given by  $\delta(x, y, z) = z$ .

**Problem 3** (Spring 2011). Compute the work done by the force field

$$\vec{F} = (6xy - y^3)\vec{i} + (3y^2 + 3x^2 - 3xy^2)\vec{j}$$

along the path  $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$  for  $-\pi/2 \leq t \leq \pi/2$ .

**Problem 4** (Spring 2011). Evaluate

$$\int_C xy^3 dx + 3x^2 y^2 dy$$

where  $C$  is the boundary of the region in the first quadrant enclosed by the  $x$ -axis, the line  $x = 1$  and the curve  $y = x^3$ , traveled counter-clockwise.

**Problem 5** (Fall 2010). Evaluate

$$\int_C x^2 dx + y^2 dy + z^2 dz$$

where  $C$  is the straight line segment from  $(1, 2, 3)$  to  $(2, 3, 4)$ .

**Problem 6** (Spring 2011). Compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

of the vector field

$$\vec{F} = \langle 2xy^2 + 3xz^2, 2x^2y + 2y, 3x^2z - 2z \rangle$$

on the curve  $C$  given by

$$\vec{r}(t) = \langle \cos(2t) + 5 \sin(5t), 6 \sin(t) + 4 \sin(5t), \cos(2t) + \cos(5t) \rangle$$

for  $0 \leq t \leq \pi$ .

**Problem 7** (Fall 2011). Evaluate the integral

$$\int_C (y + \sin(e^{x^2}))dx - 2xdy,$$

where  $C$  is the circle  $x^2 + y^2 = 1$  traveled counter-clockwise.

**Problem 8** (Spring 2010). Evaluate

$$\int_C (6y + x)dx + (y + 2x)dy$$

where  $C$  is the circle

$$(x - 2)^2 + (y - 3)^2 = 4$$

oriented counterclockwise.

**Problem 9** (Spring 2013). A particle moves along the line segments from  $(0, 0, 0)$  to  $(1, 0, 0)$  to  $(1, 5, 1)$  to  $(0, 5, 1)$  and back to  $(0, 0, 0)$  under the influence of the vector field

$$\vec{F}(x, y, z) = z^2\vec{i} + 3xy\vec{j} + 4y^2\vec{k}.$$

Find the work done.

**Problem 10** (Spring 2013). Let  $S$  be the portion of the surface  $z = xy$  lying inside the cylinder  $x^2 + y^2 \leq 1$ . Compute the surface area of  $S$ .