Math 114. Fall 2014. Practice Exam

Problem 1. Find a unit vector orthogonal to both $\langle 1,1,0\rangle$and $\langle 1,2,3\rangle$.

Problem 2. What is the distance from the origin to the plane $x+2y+3z=6$:

Problem 3. The planes $3x+2y+z=6$ and $x+y=2$ intersect in a line ℓ. Find the distance from the origin to ℓ.

Problem 4. Find the area of the triangle whose vertices are $A=(0,0,1)$, $B=(1,2,1)$ and $C=(0,0,0)$. What is the angle between the two edges AB and AC?

Problem 5. Find the point in the plane $3x+2y+z=5$ which is closest to $(0,0,0)$.

Problem 6. A missile is launched from the top of a $15m$ high cliff. The missile reaches a maximum height of $20m$ and lands $60m$ away from its initial position. Find the missiles initial velocity.

Problem 7. On a strange planet, the acceleration due to gravity is given by $\vec{a}(t)=-6t\vec{\mathbf{j}}$ m/s^2. A projectile is fired from ground level with an initial velocity of $10m/s$ in the horizontal direction and $49m/s$ in the vertical direction. How far has the projectile travelled horizontally when it strikes the ground?

Problem 8. The lines

$$\begin{aligned} x &= 3+2t & \qquad x &= -1-s \\ y &= 3+3t & \qquad y &= 8+4s \\ z &= -t & \qquad z &= 5+2s \end{aligned}$$

intersect in a point P. write down an equation for a line which is perpendicular to these lines and which passes through P.

Problem 9. Find the relation between α and β such that the following two planes are perpendicular to each other

$$2x-y+\beta z=2014$$

$$(\alpha-3)x+11y-6z=0$$

Problem 10. A particle has an acceleration $\vec{a}(t)=t\vec{i}+t^2\vec{j}+2\vec{k}$. If its initial velocity is $v_0=\langle 1,3,7\rangle$ and it is initially at the origin, find its position function $\vec{r}(t)$.

Problem 11. Find the arc length of the curve

$$\mathbf{r}(t)=\langle t^2,\cos t+t\sin t,\sin t-t\cos t\rangle$$

for $0\leq t\leq\sqrt{2}$.

Problem 12. Find the arc length of the curve

$$\vec{r}(t) = (\cos t)^3 \vec{j} + (\sin t)^3 \vec{k}$$

between $t = 0$ and $t = 4$.

Problem 13. Find the curvature

$$\mathbf{r}(t) = \langle t^2, \cos t + t \sin t, \sin t - t \cos t \rangle$$

when $t = 1$.

Problem 14. Find the maximum curvature of the curve $\mathbf{r}(t) = \langle t, t, t^2 \rangle$.

Problem 15. Consider the helix $\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle$, compute

a. $\vec{T}, \vec{N}$ and $\vec{B}$ at time $t = 0$;

b. The curvature κ at time $t = 0$;

c. a_T and a_N where $\vec{a} = a_T \vec{T} + a_N \vec{N}$ at time $t = 0$.