Math 601 Spring 2015

Homework 11

Due Thursday April 23 at 1pm in Chris Hays mailbox

(1) Let M be a 3-dimensional non-orientable topological manifold. Show that $H_1(M, \mathbb{Z})$ is infinite.

(2) Let M be an n-dimensional non-orientable topological manifold. Show that $H_c^n(M, \mathbb{Z}) = \mathbb{Z}_2$.

(3) Show that $\mathbb{S}^2 \times \mathbb{S}^2$, $\mathbb{CP}^2 \# \mathbb{CP}^2$ and $CP^2 \# \mathbb{CP}^2$ all have the same cohomology groups, but different homotopy type by determining the intersection form.

(4) Show that a compact orientable manifold of dimension $4n + 2$ has even Euler characteristic. What can you say for other dimensions?

(5) Let X be a 4k-dimensional compact orientable topological manifold. Show that the signature has the same parity as the Euler characteristic.