Math 501 Spring 2016

Homework 9

Due: Thursday March 31 at the end of class.

(1) Shifrin p. 89 # 1

(2) Shifrin p. 89 # 2

(3) Shifrin p. 89 # 3

(4) Shifrin p. 90 # 9

(5) Shifrin p. 90 # 11

(6) Shifrin p. 91 # 15

(7) (Extra Credit)

Lat week in the extra credit problem we discussed the metric $ds^2 = \frac{4(dx^2+dy^2)}{1-(x^2+y^2)}$ on the unit ball $x^2 + y^2 < 1$ and you showed that the following curves are geodesics:

$$c(t) = (1-a)e^{i\alpha t} + ae^{-i\beta t} \text{ where } \alpha = \frac{a}{2\sqrt{a(1-a)}},\ \beta = \frac{1-a}{2\sqrt{a(1-a)}}$$

(a) Show that if such a geodesic is closed, then it must have length $4\pi\sqrt{n}$ for some $n \in \mathbb{Z}$; i.e., find the values of $a, 0 < a < 1$ where c closes up to form a closed geodesic.

(b) Show that the number of closed geodesics of this length $4\pi\sqrt{n}$ is equal to $\psi(n)$. Here $\psi(n)$ is a number theoretic function which is equal to the number of ways you can write $n = pq$ where $p, q \in \mathbb{Z},\ p \leq q$ and p and q have no common divisor.

(c) Draw some pictures of these closed geodesics in a graphics program.