Math 501 Spring 2016

Homework 3

Due: Thursday February 11 at the end of class. A portion of the homework will be graded (by Anusha Krishnan) and returned to you at the end of the next class. Remember to staple your homework and put your name on it.

(1) Let α be a closed convex regular curve in the plane with normal n. The curve $\beta(s) = \alpha(s) - rn(s)$, r fixed, is called a parallel curve to α.
   (a) Show that $L(\beta) = L(\alpha) + 2\pi r$.
   (b) Show that if κ_1 is the curvature of α, and κ_2 is the curvature of β, then $\kappa_2 = \frac{\kappa_1}{1+r\kappa_1}$.
   (c) For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, determine for what values of r the curve β is still regular, and draw some pictures.

(2) Show that a convex regular plane curve has no self intersections.

(3) Shifrin p. 41 Problem 1

(4) Shifrin p. 41 Problem 3

(5) Shifrin p. 42 Problem 11

(6) (Extra Credit)
   (a) Let α be a simple closed regular curve in the plane. Assume that its curvature satisfies $0 < \kappa \leq r$. Show that its length is at least $\frac{2\pi}{r}$.
   (b) Let α be a closed regular curve in the plane. Assume that its rotation index is equal to N where $N \in \mathbb{Z},\ N > 0$. Show that its length is at least $\frac{2\pi N}{r}$.