Math 501 Spring 2016

Homework 1

Due: Thursday January 28 at the end of class. A portion of the homework will be graded (by Anusha Krishnan) and returned to you at the end of the next class. Remember to staple your homework and put your name on it.

(1) Compute the curvature of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. What is the maximum and minimum and where is it assumed.

(2) Let $\alpha\colon [a,b] \to \mathbb{R}^2$ be a curve and β a circle of radius r. Assume that α lies inside β but touches it at a point $\alpha(t_0)$ with $a < t_0 < b$. Show that $k \geq \frac{1}{r}$ where k is the curvature of α at $\alpha(t_0)$.

Hint: Use the fact that the curvature of a curve is invariant under translation and rotation in order to simplify your picture. Or see Shifrin p. 18 Problem 7

(3) Shifrin p. 18 Problem 4.

(4) Shifrin p. 19, Problem 11

(5) Shifrin p. 10, Problem 13, parts a) - d).

(6) (Extra Credit) Problem 10, p. 9 in Shifrin. Turn in Extra Credit problems on a separate sheet of paper.