Math 501 Spring 2016

Practice Exam

You should regard this as a practice exam for the Final (longer than the actual Final will be). It does not have to be turned in (except for Extra Credit Problems), but you can ask me questions about problems during office hour or via e-mail.

The Final Exam will be in class April 26. You can use the book that you studied from (for most of you Ted Shifrin's notes, but any other book can be used as well) during the exam. One page of notes with basic formulas, hand written, no copies, can be used as well.

(1) Compute the curvature and torsion of the curve

$$\alpha(t) = \Big(t - \ln(t+1),\ 2\sqrt{2}\left(\sqrt{t} - \arctan(\sqrt{t}\,)\right),\ \ln(t+1)\Big)$$

(2) Compute the sectional curvature and mean curvature of a hyperboloid.

(3) Compute the shape operator and the principal curvatures of the surface $x = y + yz + 1$.

(4) Show that for every compact surface $M$, the Gauss map $N\colon M \to \mathbb{S}^2(1)$ is onto.

(5) Show that there is no compact minimal surface in $\mathbb{R}^3$.

(6) Consider the surface $M^2$ defined as the graph $z = f(x, y)$ and $\alpha(t) = (x(t), y(t), f(x(t), y(t))$ a curve on this surface. Derive a formula for the geodesic curvature of $\alpha$ as a curve on $M^2$.

(7) Show that an isometry takes geodesics to geodesics

(8) Compute the Euler characteristic of real projective space.

(9) Show that for a surface $M^2$ in $\mathbb{R}^3$, homeomorphic to a sphere, $\int_M H^2 dA \geq 4\pi$, with equality if and only if $M$ is a round sphere.

(10) (Extra Credit 1) Show that for a closed curve on the unit sphere, parametrized by arclength, the integral of the torsion is 0.

(11) (Extra Credit 2) Use a graphics program to draw a picture of the analytic flat Moebius strip by Schwarz (paper on my home page).

(12) (Extra Credit 3) Ted Shifrin asked me to let him know about any mistakes, corrections, or comments about the notes or assigned problems. Give me a list, and I will pass them along.