Math 501 Spring 2016

Homework 10

Due: Thursday April 14 at the end of class.

(1) Shifrin p. 98 # 8

(2) Let $\mathbb{H}^2 = \{(x,y) \in \mathbb{R}^2 \mid y > 0\}$ with $ds^2 = \frac{dx^2+dy^2}{y^2}$ be the upper half space model and $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 1\}$ with $ds^2 = \frac{4(dx^2+dy^2)}{(1-x^2-y^2)^2}$ the disk model of hyperbolic space.
  (a) Show that the Cayley transformation $T \colon D \to \mathbb{H}$ defined by $T(z) = -i\frac{z+i}{z-i}$ is an isometry.
  (b) Show that boundary of D is infinite distance away from the origin.

(3) Consider the circle C: $x^2+y^2 = R^2$ with $R < 1$ in the disc module D.
  (a) Show that all points of C have constant distance from the origin, and compute that distance r (i.e. C is a geodesic circle of radius r in the hyperbolic metric).
  (b) Compute the hyperbolic area inside C.
  (c) Comupte the geodesic curvature of C.
  You may want to use some "obvious" isometries of D.

(4) (Extra Credit) Show that the geodesics in the disc model D are circles orthogonal to the boundary.