12.1

Three-Dimensional Coordinate Systems

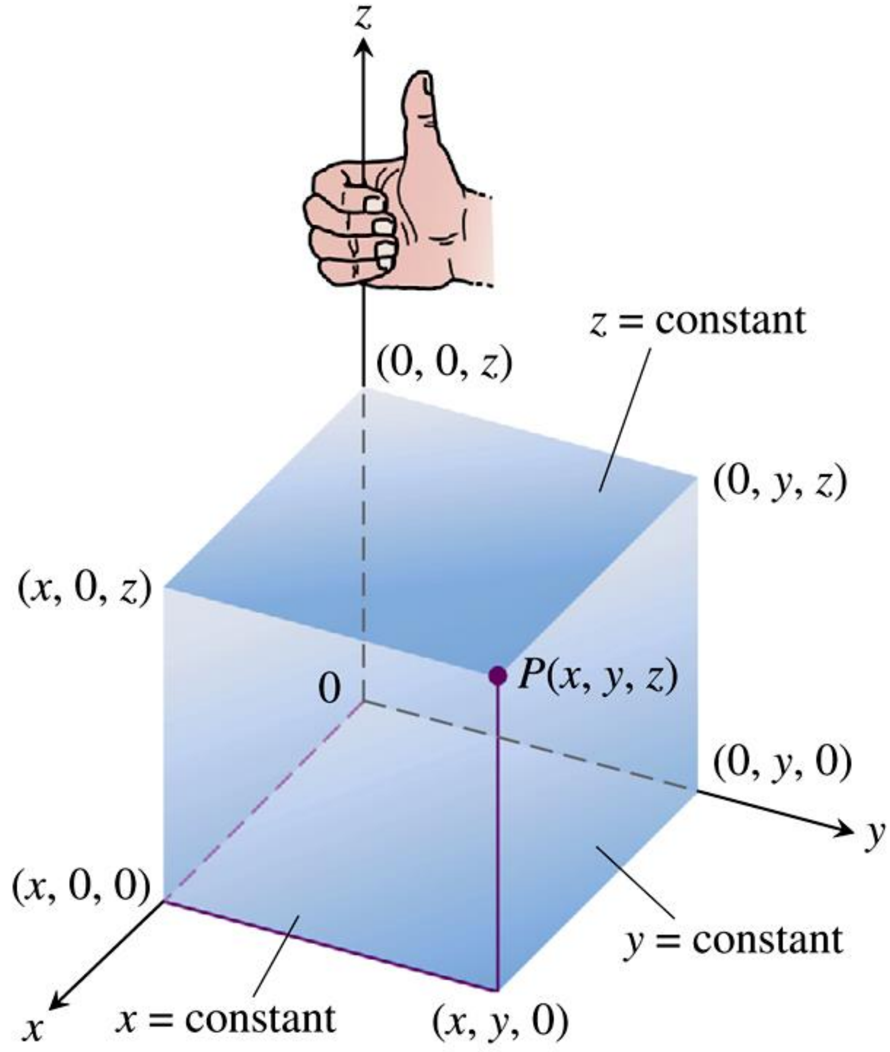

FIGURE 12.1 The Cartesian coordinate system is right-handed.

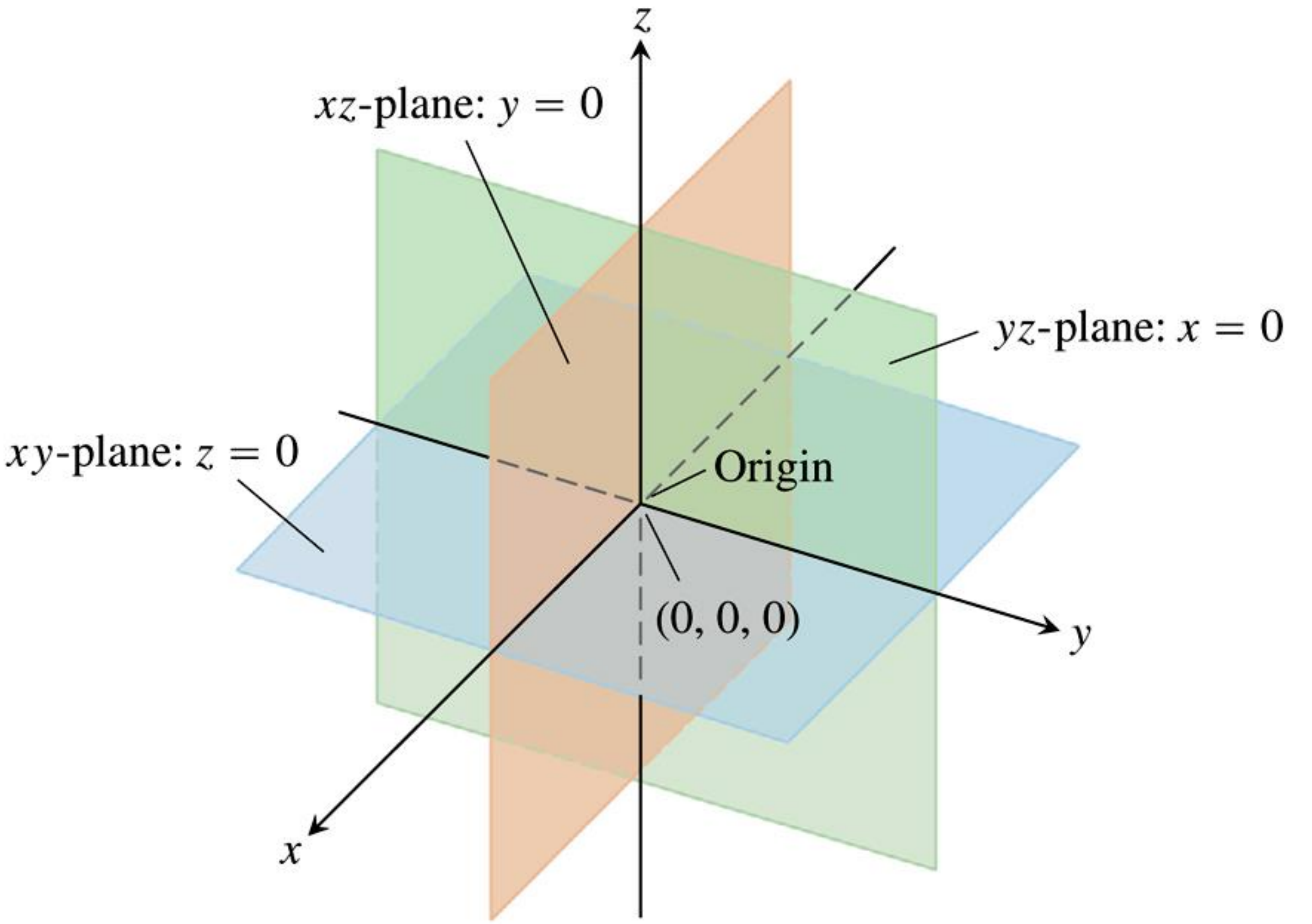

FIGURE 12.2 The planes $x = 0$, $y = 0$, and $z = 0$ divide space into eight octants.

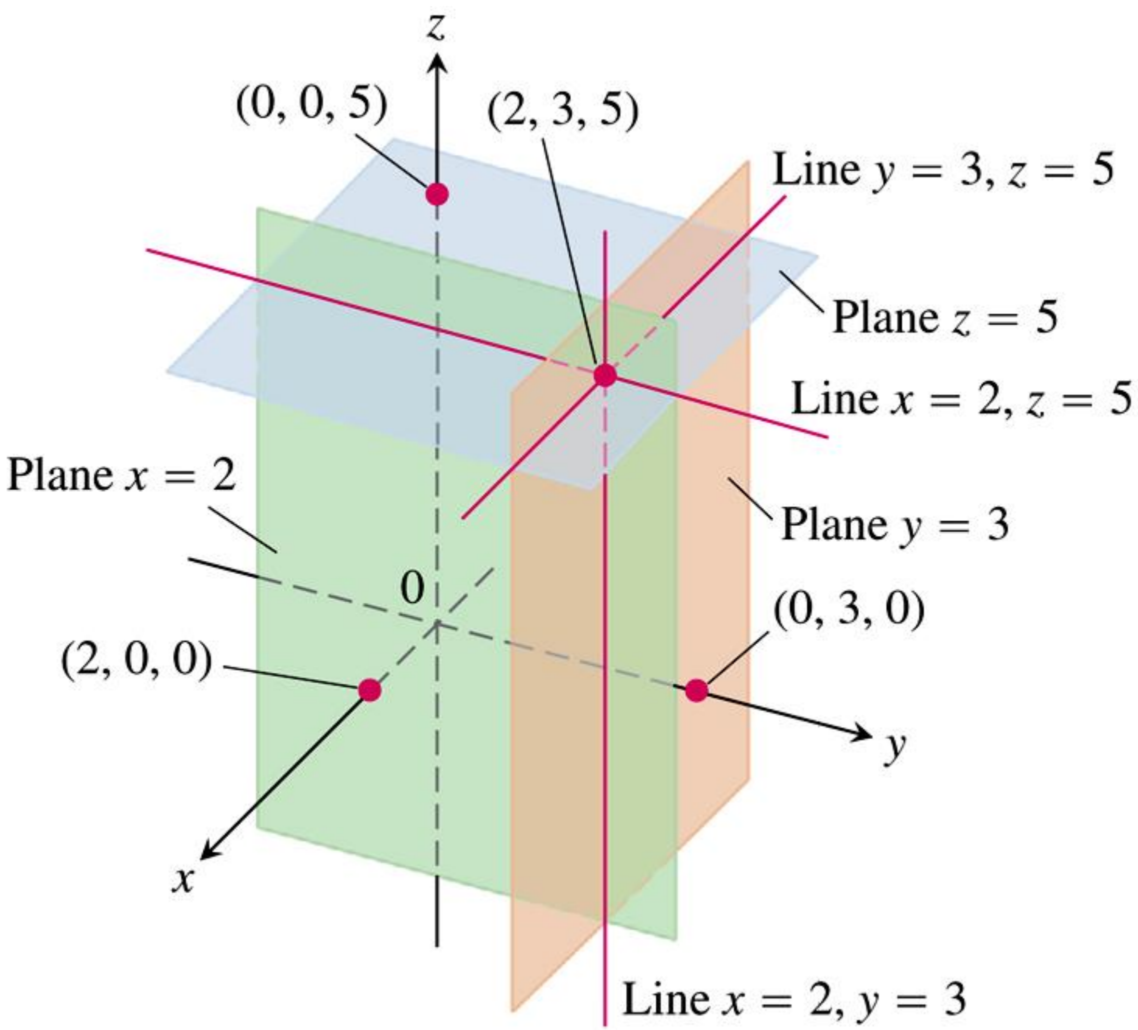

FIGURE 12.3 The planes $x = 2, y = 3$, and $z = 5$ determine three lines through the point (2, 3, 5).

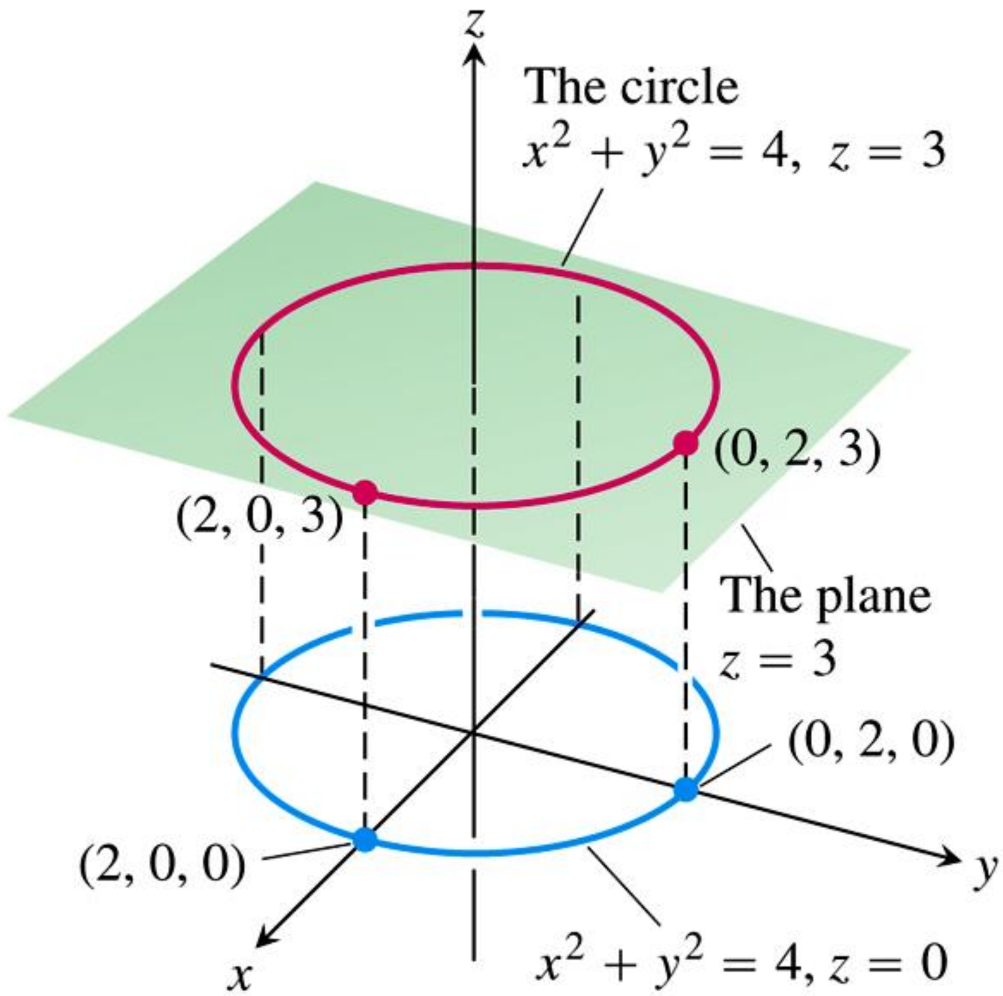

FIGURE 12.4 The circle $x^2 + y^2 = 4$ in the plane $z = 3$ (Example 2).

The Distance Between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

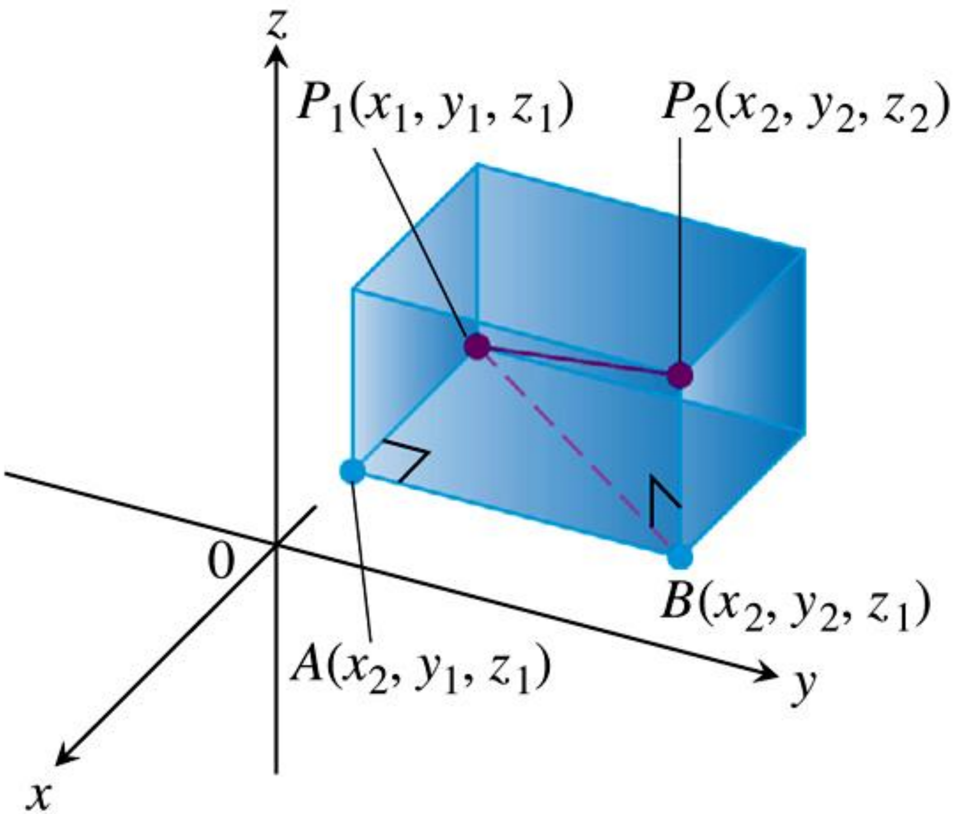

FIGURE 12.5 We find the distance between P_1 and P_2 by applying the Pythagorean theorem to the right triangles P_1AB and P_1BP_2.

EXAMPLE 3 The distance between $P_1(2, 1, 5)$ and $P_2(-2, 3, 0)$ is

$$\begin{aligned} |P_1P_2| &= \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2} \\ &= \sqrt{16 + 4 + 25} \\ &= \sqrt{45} \approx 6.708. \end{aligned}$$

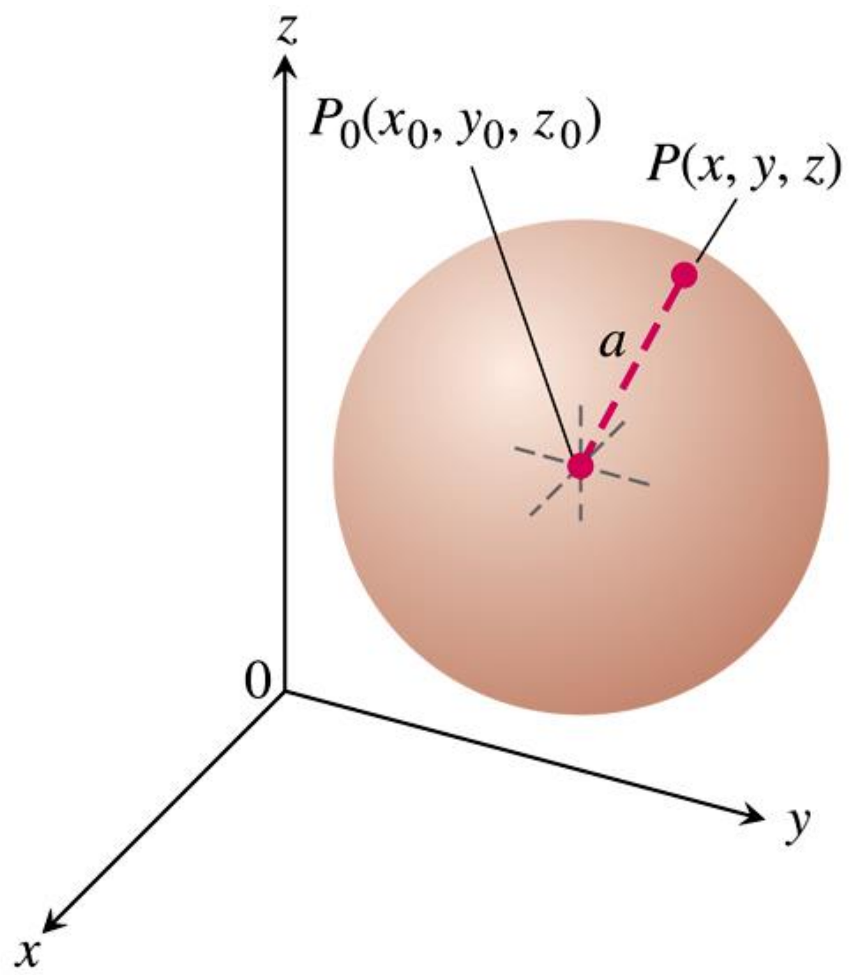

FIGURE 12.6 The sphere of radius a centered at the point (x_0, y_0, z_0).

EXAMPLE 4 Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0.$$

Solution We find the center and radius of a sphere the way we find the center and radius of a circle: Complete the squares on the x-, y-, and z-terms as necessary and write each quadratic as a squared linear expression. Then, from the equation in standard form, read off the center and radius. For the sphere here, we have

$$\begin{aligned}
x^2 + y^2 + z^2 + 3x - 4z + 1 &= 0 \\
(x^2 + 3x) + y^2 + (z^2 - 4z) &= -1 \\
\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + y^2 + \left(z^2 - 4z + \left(\frac{-4}{2}\right)^2\right) &= -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2 \\
\left(x + \frac{3}{2}\right)^2 + y^2 + (z - 2)^2 &= -1 + \frac{9}{4} + 4 = \frac{21}{4}.
\end{aligned}$$

From this standard form, we read that $x_0 = -3/2$, $y_0 = 0$, $z_0 = 2$, and $a = \sqrt{21}/2$. The center is $(-3/2, 0, 2)$. The radius is $\sqrt{21}/2$.