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Note

A Note On $P(-\lambda; G)$

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We note here a simple combinatorial interpretation of the values of the chromatic polynomial at negative integers. If

$$P(\lambda, G) = \sum_{j=0}^{n-1} (-1)^j a_j \lambda^{n-j} \qquad (a_0 = 1),$$

then a well-known theorem of Whitney asserts that a_j is the number of j subsets of edges of G which contain no broken circuit (cf. [1] for terminology and proof).

Now suppose that $\lambda + 1$ colors are available, one of which is blue. In how many ways can we color the *edges* of G with no blue broken circuit (i.e., no broken circuit all of whose edges are blue)?

Choose a set S of edges which contains no broken circuit, and color its edges blue. If $|S| = j$, there are a_j choices for S, and the remaining $|E(G)| - j$ edges of G can be colored in λ^{E-j} ways, so the number of such colorings is

$$\sum_{j=0}^{n-1} a_j \lambda^{E-j} = \lambda^{E-n} \mid P(-\lambda; G) \mid$$

and we have the

THEOREM $|P(-\lambda; G)|$ *is* λ^{n-E} *times the number of* $(\lambda + 1)$ *colorings of the edges of G which contain no blue broken circuit.*

REFERENCE

1. O. ÖRE, "Theory of Graphs," A.M.S. Colloquium Publication No. 38, Providence, R. I., 1962.