MATH 603: HOMEWORK #4

DUE IN SIGGY MOORE'S MAILBOX BY WEDNESDAY, MARCH 15, 2017

1. Straightedge and compass constructions.

1. Do problem # 2 of section 13.3 of Dummit and Foote's book, without consulting any other references.
2. For each integer $n \geq 2$, describe an explicit construction by straightedge and compass of the regular 2^n-gon centered at the origin which has the point $(1, 0)$ on the x-axis as a vertex.
3. In class we discussed the Poincare disk P. As a point set, P is the interior of the unit disc about the origin in the Euclidean plane $\mathbb{R}^2$. The hyperbolic length of an arc in P is the integral of
$$\frac{|ds|}{1 - r^2}$$
over the arc, were $|ds|$ is the differential of Euclidean length and r is the distance of a point along the arc from the origin. We discussed how hyperbolic lines in P are either Euclidean lines or arcs of Euclidean circles which intersect the boundary of P at right angles. Hyperbolic circles are Euclidean circles with centers which will in general be different from the Euclidean center. Starting with the origin $(0, 0)$ in P and the point $(1/2, 0)$, find the next three points which can be constructed by hyperbolic straightedge and compass constructions.

2. Splitting fields, separability and normality.

1. Find the splitting field E of $f(x) = x^6 + x^3 + 1$ over $\mathbf{Q}$ inside $\mathbf{C}$, and determine the degree $[E : \mathbf{Q}]$.
2. Let F be a field characteristic $p > 0$, and suppose that α is algebraic over F. Show that α is separable over F if and only if $F(\alpha) = F(\alpha^{p^n})$ for all integers $n > 0$.
3. A field F is called perfect if either $char(F) = 0$ or $char(F) = p$ and the Frobenius map $\Phi : F \to F$ defined by $\Phi(\alpha) = \alpha^p$ is an isomorphism. Show that a field F is perfect if and only if every algebraic extension of F is separable.
4. Suppose F is a field, $f(x)$ is a monic irreducible polynomial in $F[x]$ and that K is a finite normal extension of F. Suppose that $g(x)$ and $h(x)$ are monic irreducible factors of $f(x)$ in $K[x]$. Show that there is an automorphism σ of K over F such that $\sigma(g(x)) = h(x)$, where $\sigma(g(x))$ is the polynomial which results from applying σ to the coefficients of $g(x)$. Give an example in which this is not true if K is not normal over F.