# MATH 350: HOMEWORK #5

DUE IN LECTURE WEDNESDAY, OCT. 29, 2014.

## 1. Primitive roots and their applications

1. Suppose that $p$ is an odd prime and $c$ is a positive integer. Prove that if $b$ is an integer which is congruent to 1 mod $p$, then an odd power of $b$ is congruent to 1 mod $p^c$. (Hint: Show that the odd power can be taken to be a power of $p$.)
2. Suppose $c$ is a positive integer and $n = p^c$ for some odd prime p. Using the preceding problem and the fact that n has a primitive root, show that an integer $d$ which is prime to $p$ is a square mod $n = p^c$ if an only if $d$ is a square mod $p$.
3. Do problem 13 of section 9.3 of Rosen's book. You can use what we showed in class about which integers are primitive roots.
4. Do problem 2 of section 9.4 of Rosen's book.
5. Do problem 2 of section 9.5 of Rosen's book.

## 2. Beginning of quadratic reciprocity

6. Do problem 5 of section 11.1 of Rosen's book.
7. Show that if $p$ is an odd prime, then the product of all the non-zero quadratic residues mod $p$ is congruent to 1 mod $p$ if $p \equiv 3 \bmod 4$ and is congruent to $-1$ mod $p$ if $p \equiv 1 \bmod 4$. (Hint: Let $x$ be a primitive root, and show that the non-zero quadratic residues are the set $\{x^2, x^4, \ldots, x^{p-1}\}$ where $x^{p-1} \equiv 1 \bmod p$.)
8. Suppose $p$ is a prime and that $p \equiv 3 \bmod 4$. We showed in class that $-1$ is not a square mod $p$. Show that for all non-zero residue classes $y$ mod $p$, either $y$ is a quadratic residue or $-y$ is a quadratic residue. Use this and problem #7 above to do problem 9 of section 11.1 of Rosen's book.