

MATH 350: HOMEWORK #4

DUE IN LECTURE FRIDAY, OCT. 17, 2014.

1. p -ADIC NUMBERS AND HENSEL'S LEMMA

1. How many solutions $x \bmod 7$ are there to the congruence $x^2 + 4x + 2 \equiv 0 \pmod{7}$? How many solutions x in the 7-adic numbers \mathbb{Z}_7 are there to the equation $x^2 + 4x + 2 = 0$?
2. Do problem 12 on page 174 of Rosen's book. Then show that with the notations of this problem, there is a element x of the p -adic integers \mathbb{Z}_p such that $f(x) = 0$ and $x \equiv a \pmod{p^{k-j}\mathbb{Z}_p}$. Is such an x unique?

2. EULER'S THEOREM

3. Do problem 5 of the exercises for section 6.3 of Rosen's book.
4. Do problem 11 of the exercises for section 6.3 of Rosen's book. (Hint: Raise numbers to an appropriate power to find their inverses modulo a given integer.)
5. Do problem 9 of the exercises for section 6.3 of Rosen's book.