MATH 280: HOMEWORK #1

DUE IN LECTURE ON JAN. 24, 2019.

1. Calculating the Shannon entropy of your app usage.

Both Apple and Google have apps that will tell you how many minutes you have spent each day (and also each week) on various web pages and apps on iphones and android phones. Here is a link describing these apps:

https://www.macworld.com/article/3295880/ios/android-digital-wellbeing-vs-ios-screen-time.html

For one day, find the $N = 4$ apps you have used most often. An iphone will tell you how much time you used each app; I expect an android phone will do the same. Let's assume that no two apps were being used simultaneously. If t_i is the amount of time spent using the i^{th} app, the probability of using the i^{th} app is $p_i = t_i/(t_1 + \cdots + t_N)$. List $t_1, \ldots, t_N$ (but not the names of the apps!) and then calculate the Shannon entropy

$$H(p_1, \ldots, p_N) = -\sum_{i=1}^{N} p_i \cdot \log_2(p_i).$$

Remember we are only using the $N = 4$ most used apps in order to simplify the calculations. You may find Wolfram alpha useful in doing these.

2. Properties of Shannon entropy

A probability vector $v = (p_1, \ldots, p_N)$ of length $N \geq 1$ is a vector with real components such that $0 \leq p_i \leq 1$ for all i and $p_1 + \cdots + p_N = 1$. Let $T(N)$ be the set of all such v.

The Shannon entropy $H(p_1, \ldots, p_N)$ associated to $v = (p_1, \ldots, p_N) \in T(N)$ should satisfy three axioms:

a. $H(p_1, \ldots, p_N)$ is continuous a continuous function of v.
b. Let $A(N) = H(p_1, \ldots, p_N)$ when $p_i = 1/N$ for $i = 1, \ldots, N$. Then $A(N)$ should be monotonically increasing with N.
c. Suppose $1 \leq k \leq N$, $\ell = N - k$ and that there are probability vectors $(z_1, z_2) \in T(2)$, $(d_1, \ldots, d_k) \in T(k)$ and $(e_1, \ldots, e_\ell) \in T(\ell)$ such that

$$(p_1, \ldots, p_k, p_{k+1}, \ldots, p_N) = (z_1 d_1, z_1 d_2, \ldots, z_1 d_k, z_2 e_1, z_2 e_2, \ldots, z_2 e_\ell)$$

Then
$$H(p_1, \ldots, p_N) = H(z_1, z_2) + z_1 H(d_1, \ldots, d_k) + z_2 H(e_1, \ldots, e_\ell).$$

Problem: Show that for all $K > 0$ the function

$$H(p_1, \ldots, p_N) = -K \cdot \sum_{i=1}^{N} p_i \cdot \log_2(p_i)$$

does have properties (a), (b), (c) when we define $0 \cdot \log_2(0) = 0$. (Hints: To show property (a), you will need to show that $\lim_{r \to 0^+} r \cdot \log_2(r) = 0$. For property (c), you will need to use that $z_1 + z_2 = d_1 + \cdots + d_k = e_1 + \cdots e_\ell = 1$.)