Math 210: Golden Ratio in Art

Case Study: Mondrian

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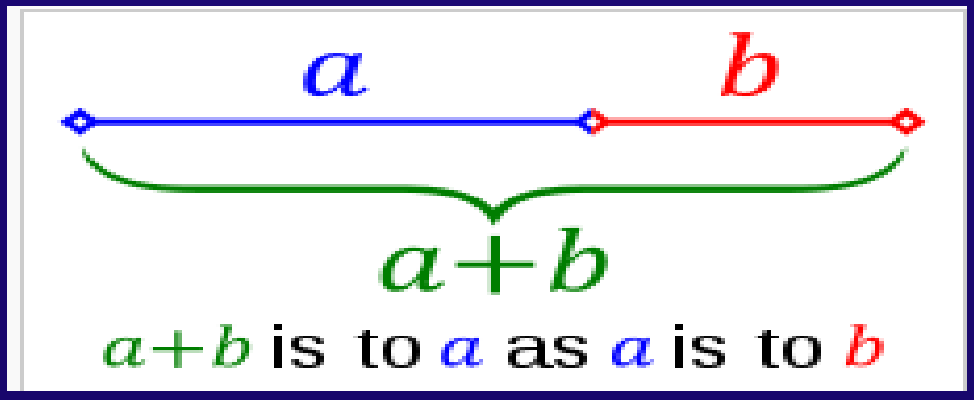
Project Goals

- Find the Golden Ratio in Art
- Determine whether or not its appearance is random or was artist's intent
- Use probability theory learned in class

Golden Ratio Summary

- Divide a line into two segments



$$\frac{a+b}{a} = \frac{a}{b} = \varphi\,.$$

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887\ldots$$

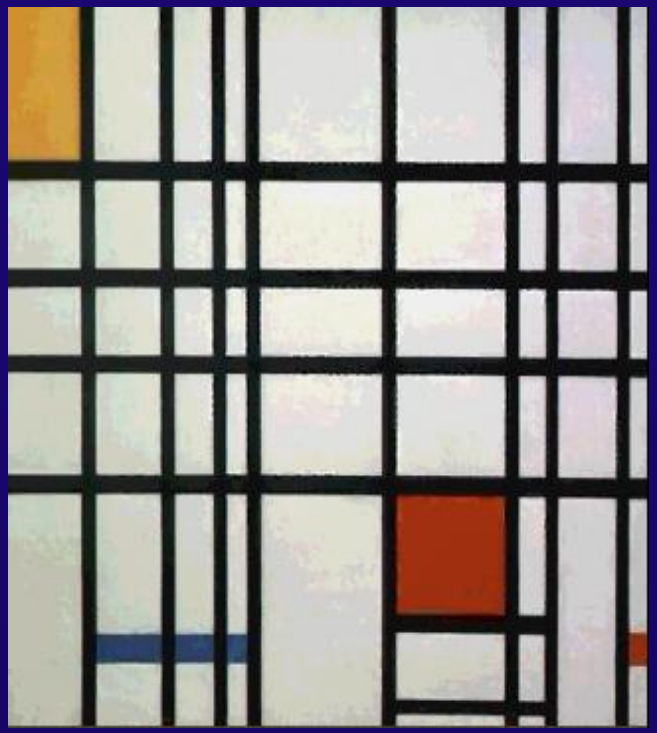
$$\varphi = [1;1,1,1,\ldots] = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}$$

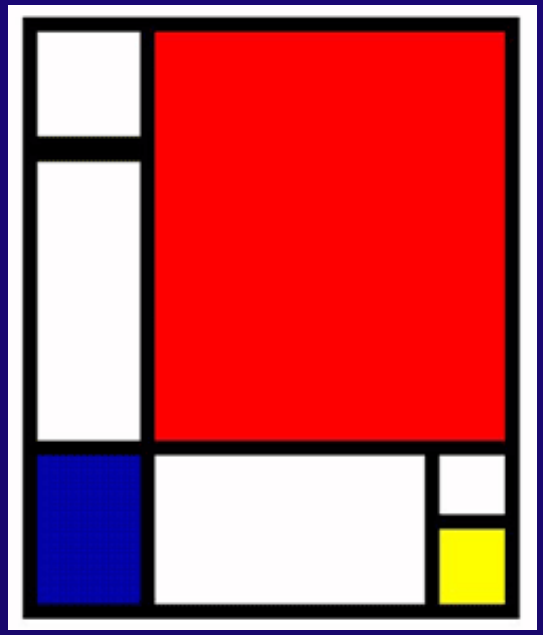
$$\varphi^{-1} = [0;1,1,1,\ldots] = 0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}$$

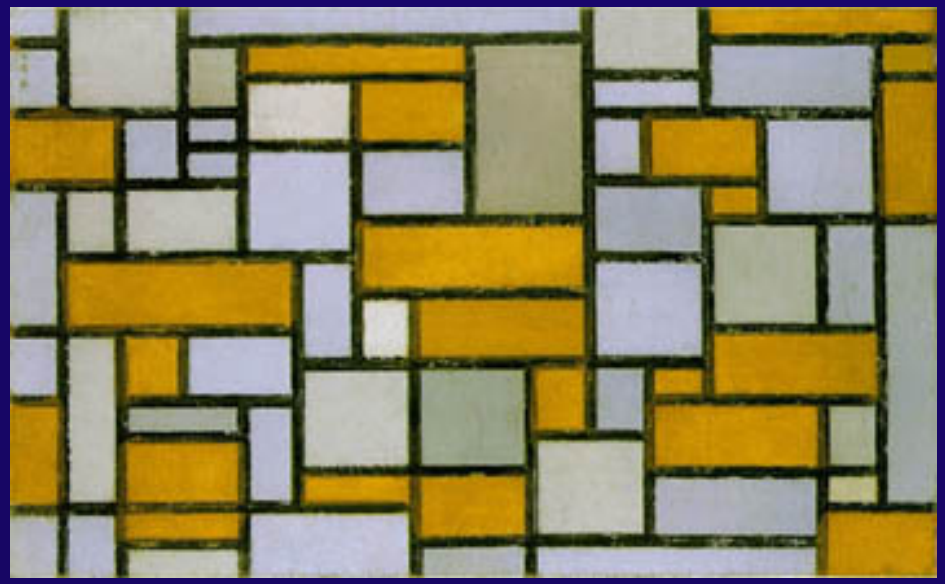
Golden Ratio In Art

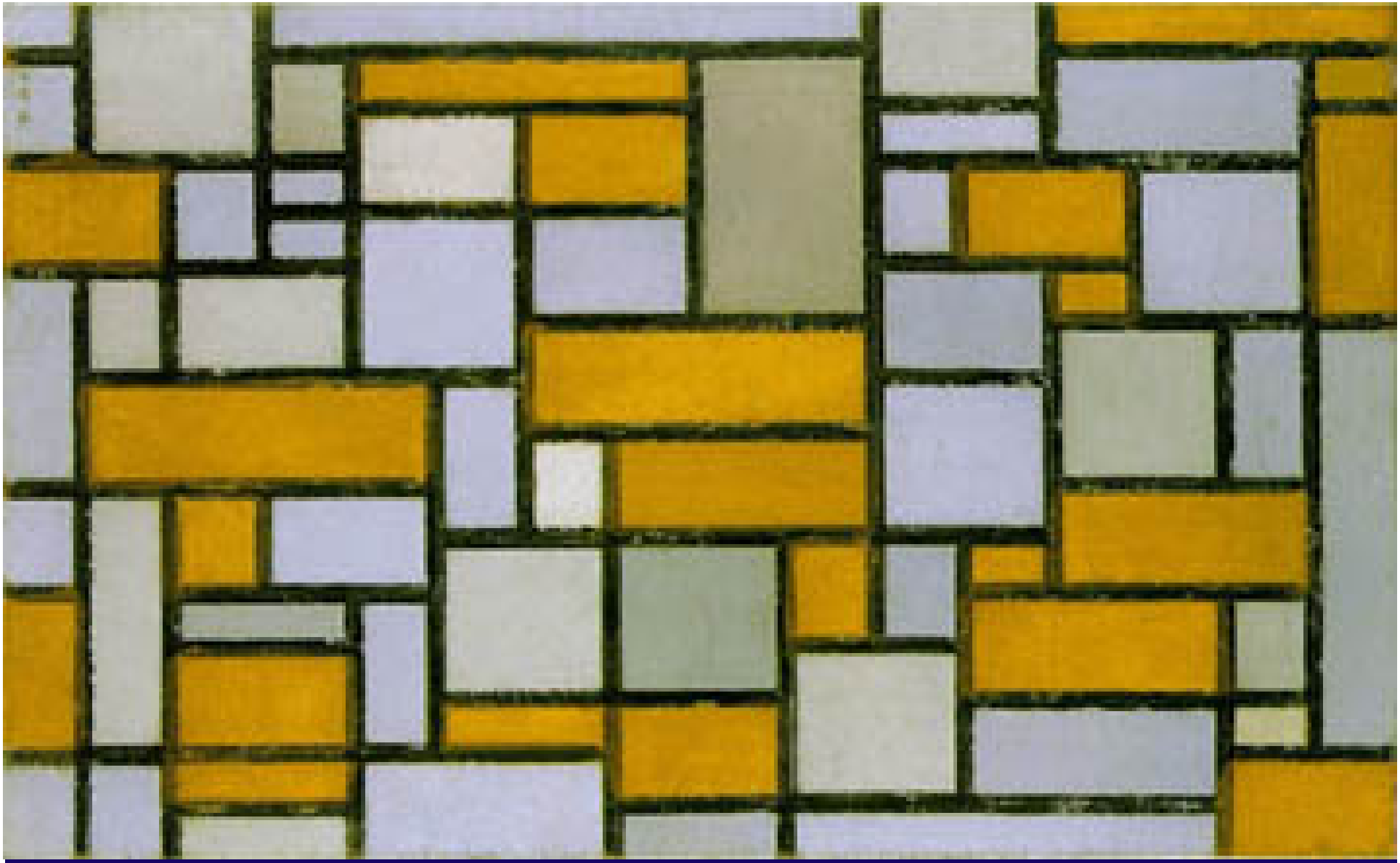
- Ancient Greeks believed most aesthetically pleasing
- Artists: Da Vinci, Dali, Mondrian
  - Piet Mondrian: Dutch, abstract artist
    - Believed mathematics and art were closely related
    - Used simple geometric shapes and primary colors (red, yellow, blue)

Mondrian

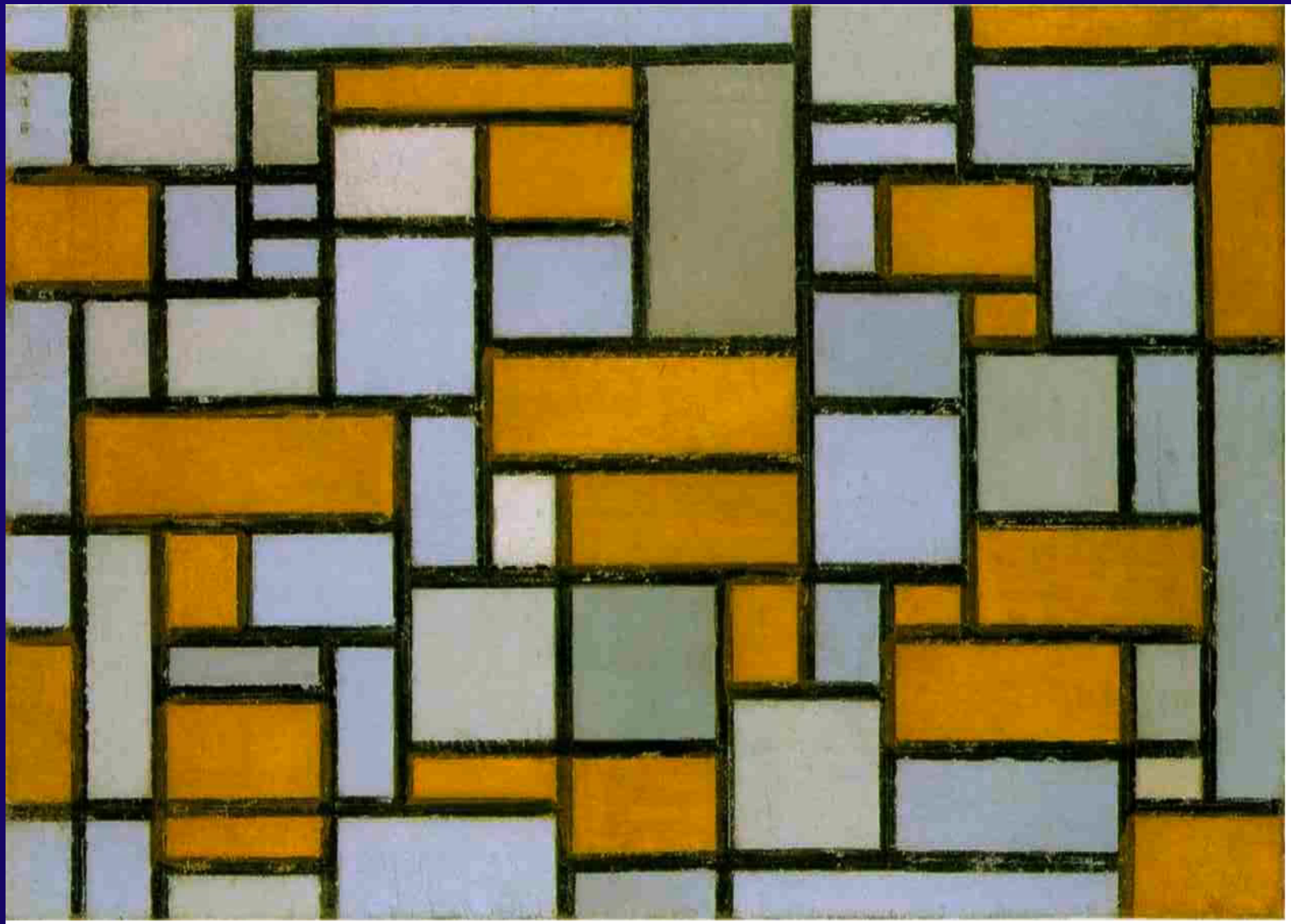




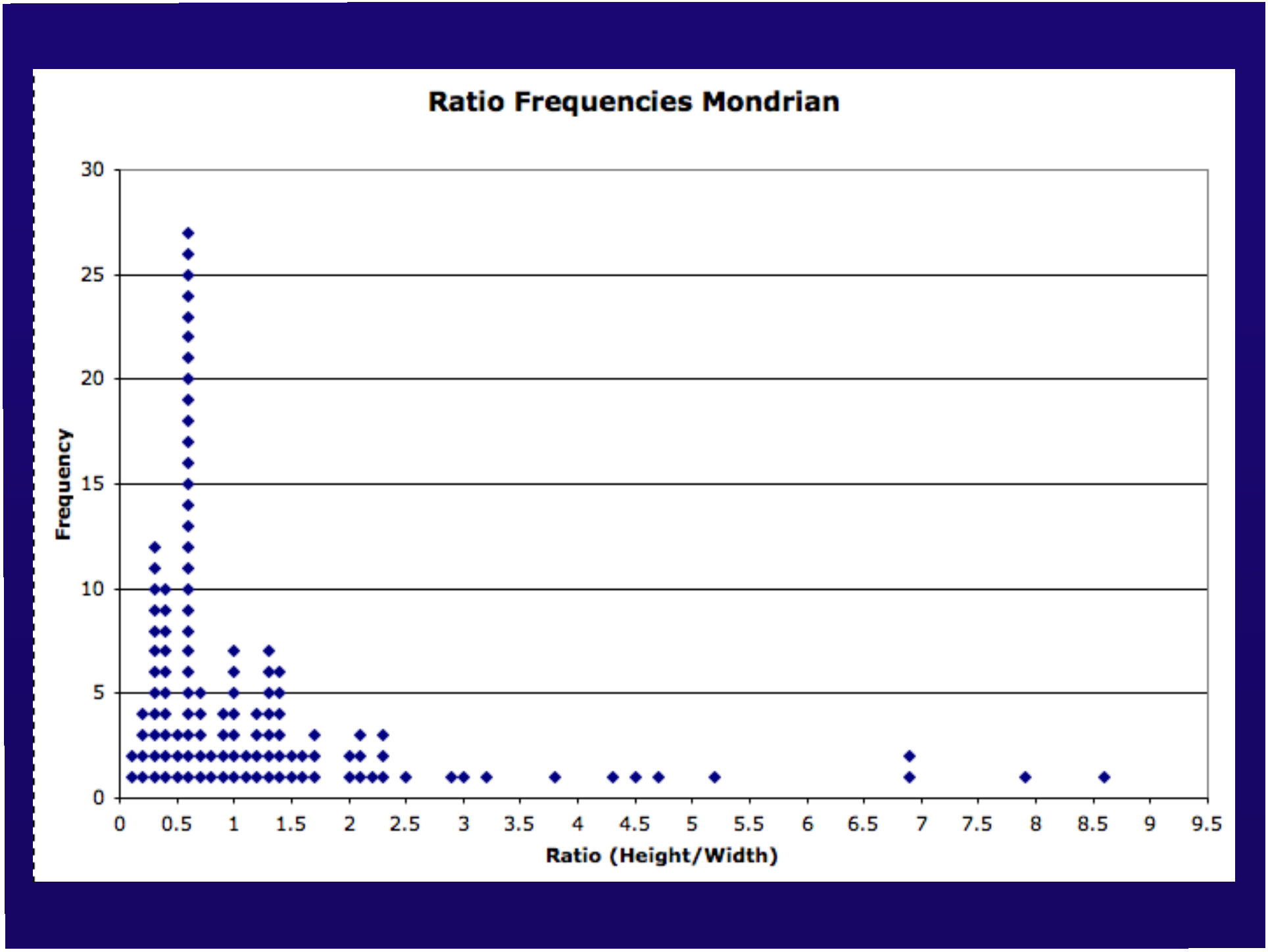


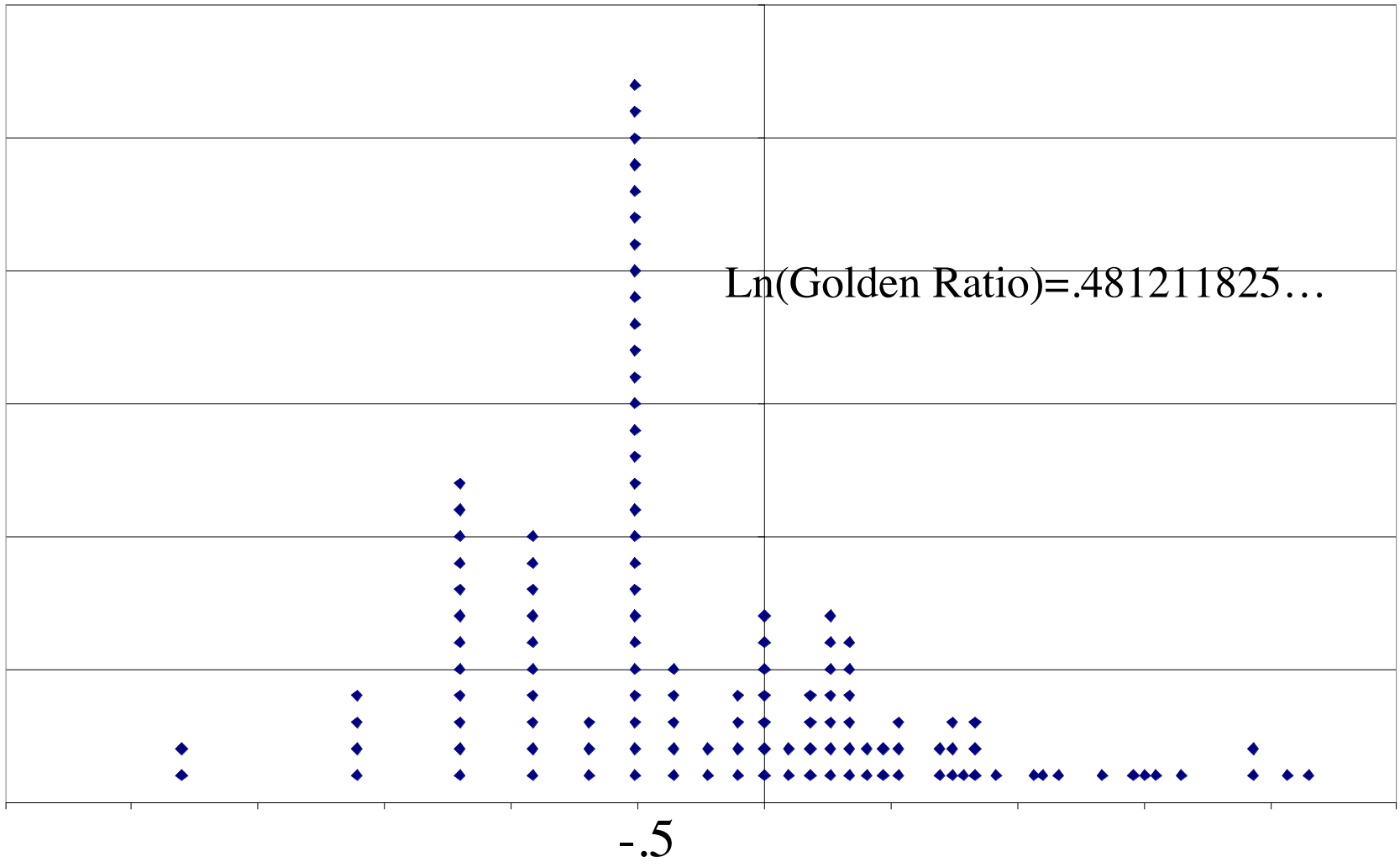


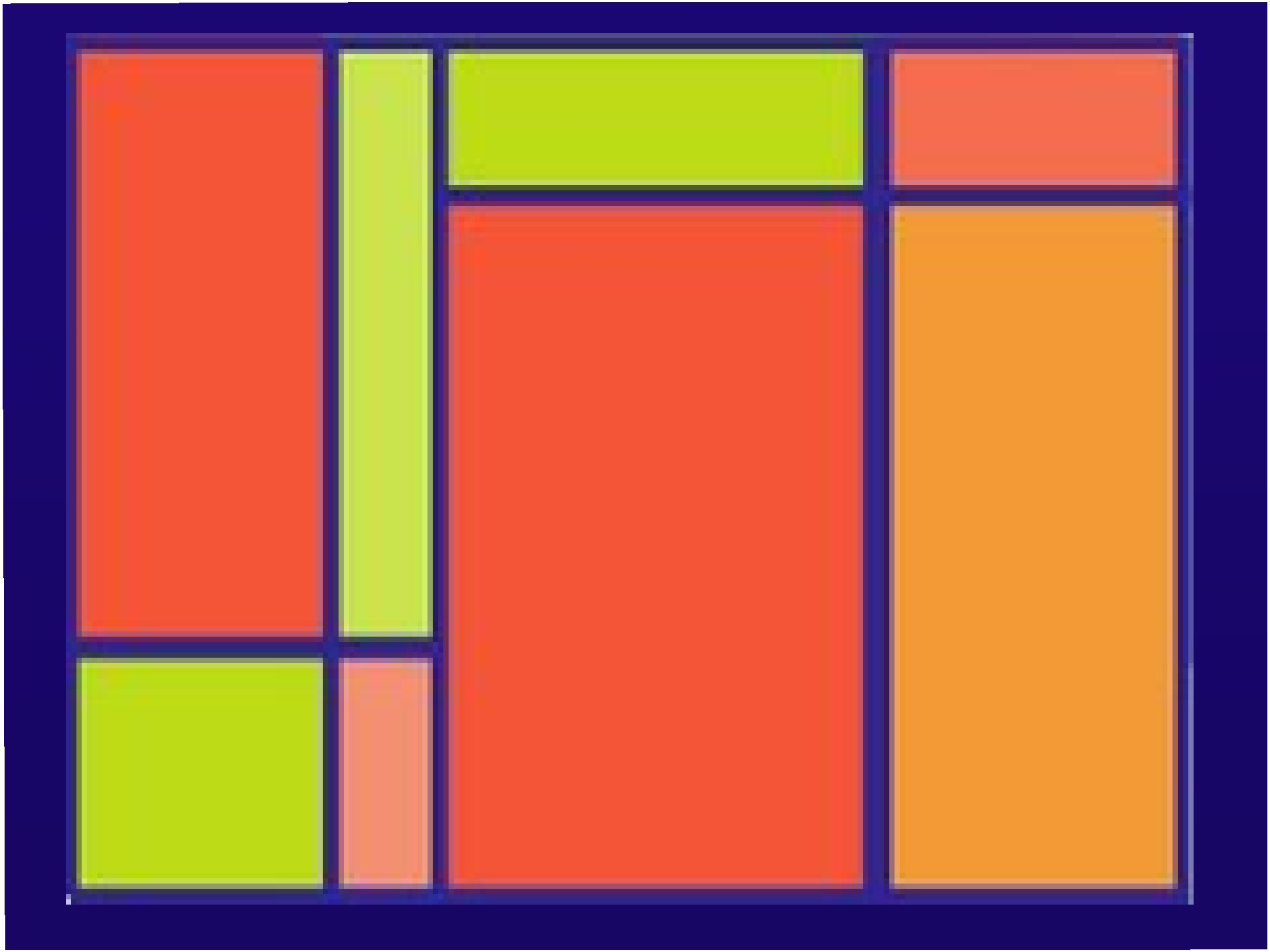
Composition with Gray and Light Brown

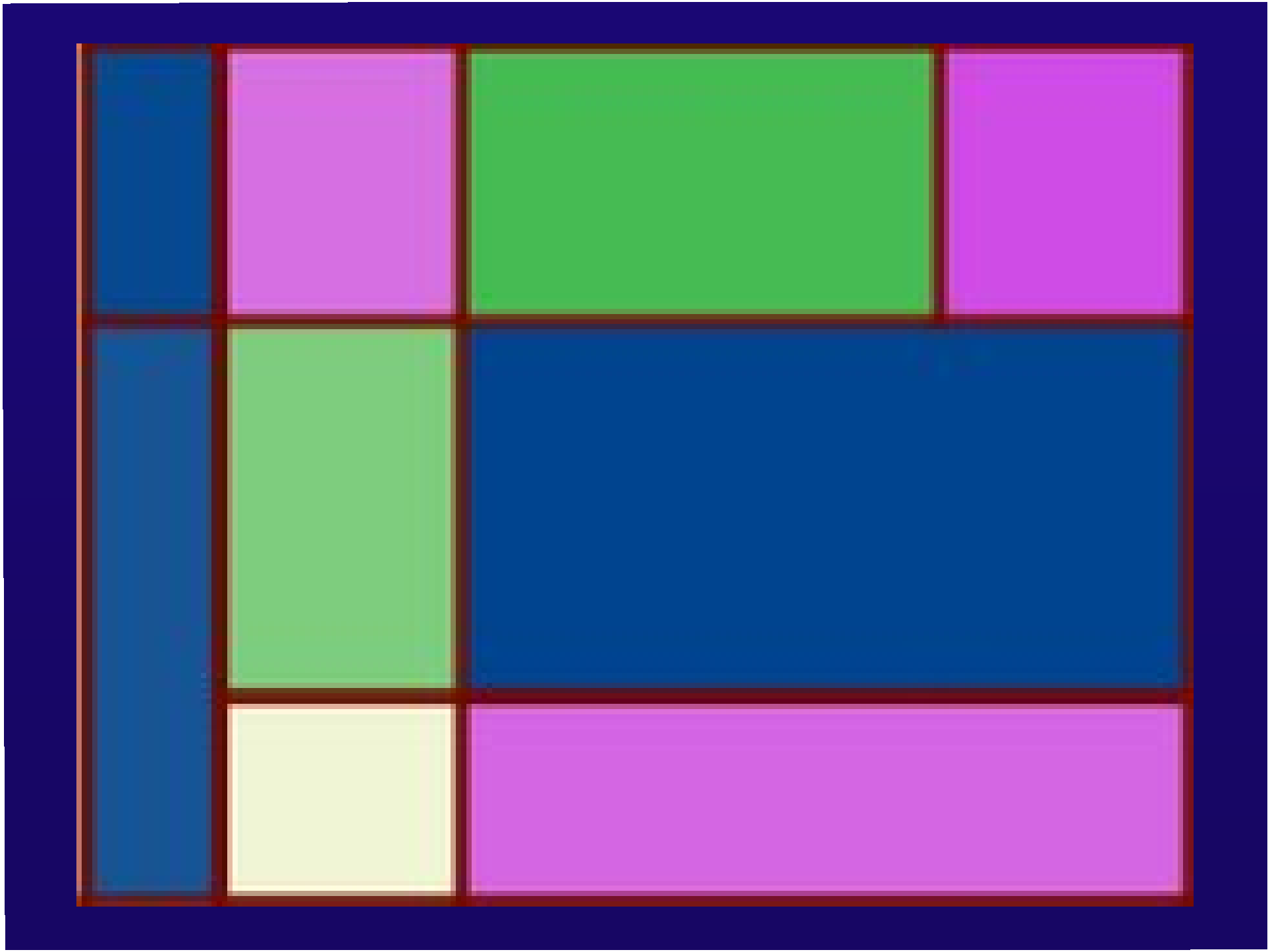


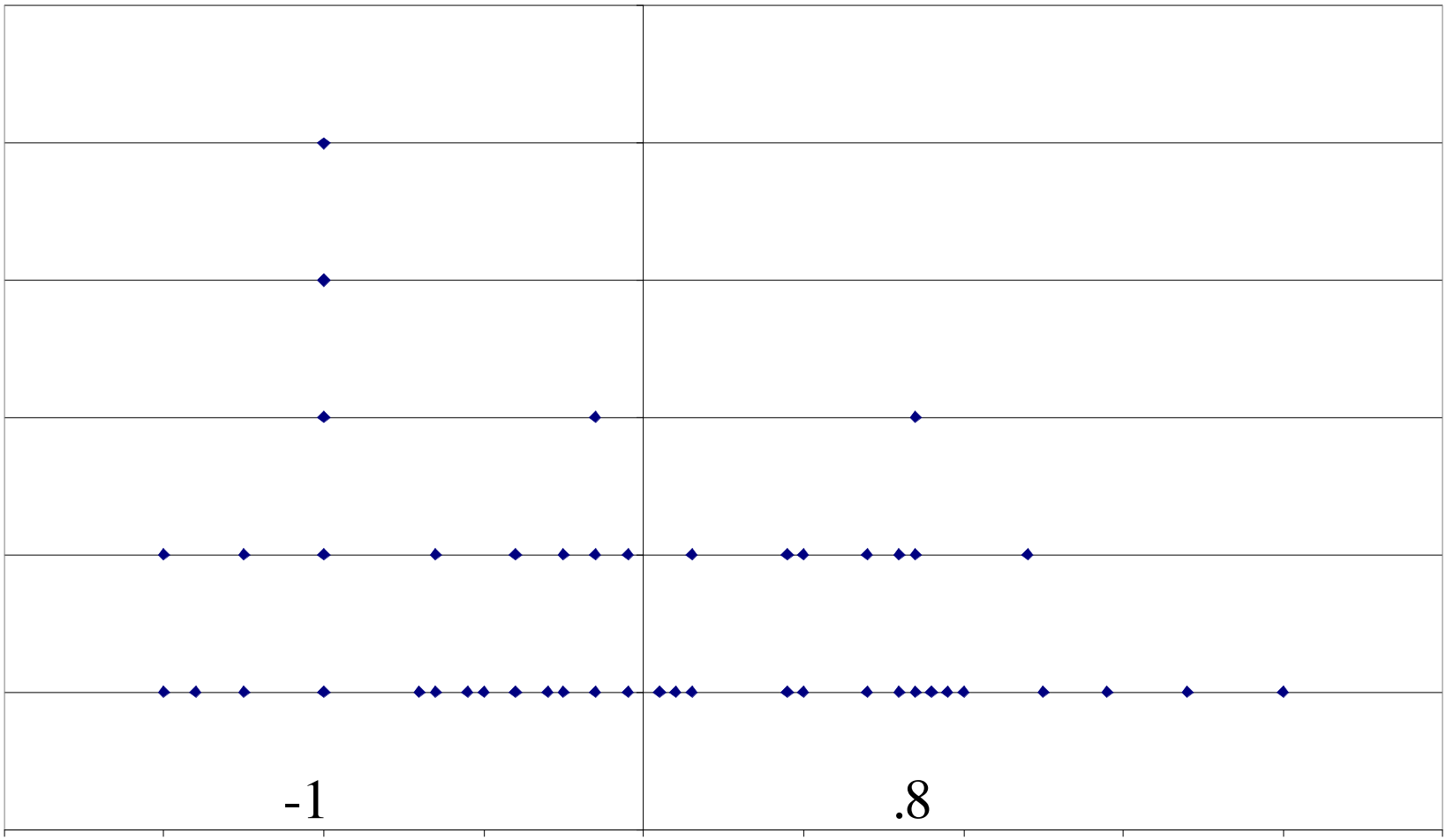
	Height	Width	H/W	W/H	Log(H/W)	Log(W/H)	Diagonal
1	0.7	1.2	0.583333	1.714286	-0.539	0.538997	1.389244
2	1.5	1.2	1.25	0.8	0.223144	-0.22314	1.920937
3	2.6	2.8	0.928571	1.076923	-0.07411	0.074108	3.820995
4	1.7	2.7	0.62963	1.588235	-0.46262	0.462624	3.190611
5	1.6	1.3	1.230769	0.8125	0.207639	-0.20764	2.061553
6	3.5	1.2	2.916667	0.342857	1.070441	-1.07044	3.7
7	1.6	1.2	1.333333	0.75	0.287682	-0.28768	2
8	1.6	2.8	0.571429	1.75	-0.55962	0.559616	3.224903
9	1.8	6	0.3	3.333333	-1.20397	1.203973	6.264184
10	1.4	1.2	1.166667	0.857143	0.154151	-0.15415	1.843909
11	2.7	1.2	2.25	0.444444	0.81093	-0.81093	2.954657
12	1.6	1.2	1.333333	0.75	0.287682	-0.28768	2
13	4.5	1.2	3.75	0.266667	1.321756	-1.32176	4.657252
14	1.6	1.2	1.333333	0.75	0.287682	-0.28768	2
15	1.6	1.4	1.142857	0.875	0.133531	-0.13353	2.126029
16	0.7	2.8	0.25	4	-1.38629	1.386294	2.886174
17	1.6	2.8	0.571429	1.75	-0.55962	0.559616	3.224903
18	0.7	2.8	0.25	4	-1.38629	1.386294	2.886174
19	0.7	2.8	0.25	4	-1.38629	1.386294	2.886174
20	1.7	4.1	0.414634	2.411765	-0.88036	0.880359	4.438468
21	2.5	1.1	2.272727	0.44	0.820981	-0.82098	2.7313
22	1.6	2.7	0.592593	1.6875	-0.52325	0.523248	3.138471
23	0.6	10.2	0.058824	17	-2.83321	2.833213	10.21763
24	1.5	1.2	1.25	0.8	0.223144	-0.22314	1.920937
25	0.6	1.2	0.5	2	-0.69315	0.693147	1.341641
26	0.7	1.2	0.583333	1.714286	-0.539	0.538997	1.389244
27	0.7	5.6	0.125	8	-2.07944	2.079442	5.64358
28	1.5	2.6	0.576923	1.733333	-0.55005	0.550046	3.001666
29	1.5	2.7	0.555556	1.8	-0.58779	0.587787	3.088689
30	2.5	2.6	0.961538	1.04	-0.03922	0.039221	3.606938
31	1.6	2.7	0.592593	1.6875	-0.52325	0.523248	3.138471
32	2.5	1.2	2.083333	0.48	0.733969	-0.73397	2.773085
33	1.5	1.2	1.25	0.8	0.223144	-0.22314	1.920937
34	2.5	2.7	0.925926	1.08	-0.07696	0.076961	3.679674
35	0.8	2.7	0.296296	3.375	-1.2164	1.216395	2.816026
	4.5	3	1.5	0.666667	0.405465	-0.40547	5.408327
37	1.7	5.7	0.298246	3.352941	-1.20984	1.209838	5.948109
38	1.5	4.2	0.357143	2.8	-1.02962	1.029619	4.459821
39	2.5	2.7	0.925926	1.08	-0.07696	0.076961	3.679674
40	1.6	2.7	0.592593	1.6875	-0.52325	0.523248	3.138471
41	0.8	2.7	0.296296	3.375	-1.2164	1.216395	2.816026
42	1.6	2.7	0.592593	1.6875	-0.52325	0.523248	3.138471
43	0.7	2.7	0.259259	3.857143	-1.34993	1.349927	2.789265


Ratio Frequencies Mondrian
Frequency
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
Ratio (Height/Width)


Ln(Golden Ratio)=.481211825…
-.5






-1
.8

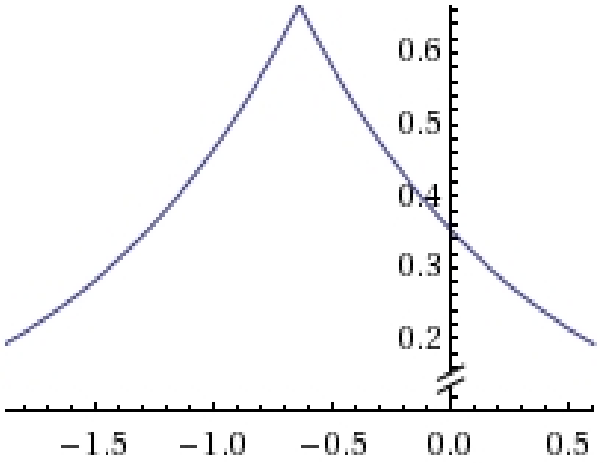
Probability Density Function for a rectangle with height to width ratio equal to 1/R

$$f_{\ln(Rr)}(u) = \frac{2e^{-|u-\ln(R)|}}{3} - \frac{e^{-2|u-\ln(R)|}}{3}$$

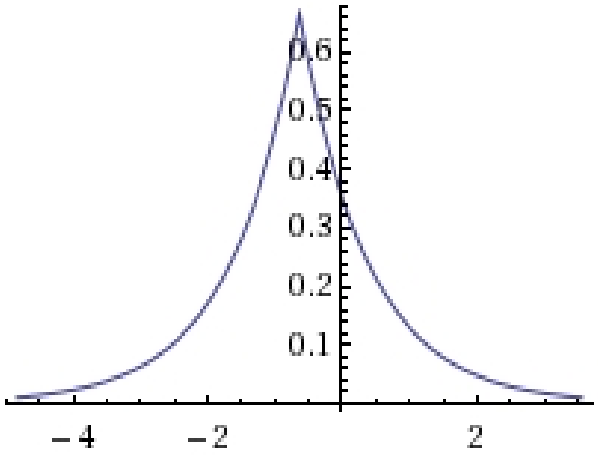
Actual Mondrian Painting: ln(R) = .63757

Random Mondrian Painting: ln(R) = 1.299

Plots:



(*u* from −1.5 to 0.5)

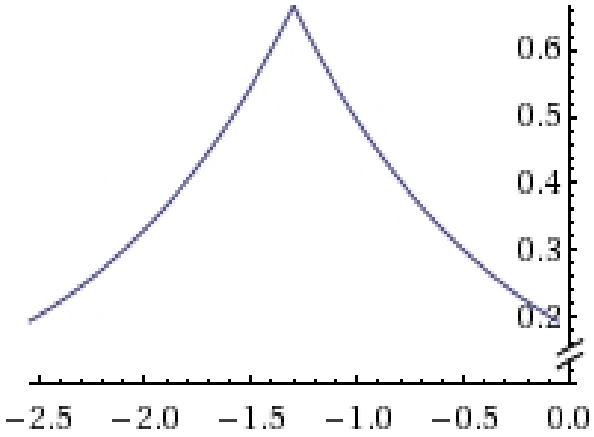


(*u* from −4 to 2)

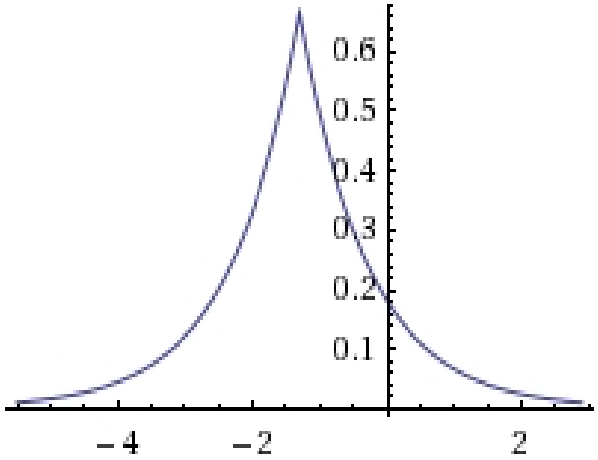
y = \frac{2e^{−|u+.637577|}}{3} −\frac{e^{−2|u+.637577|}}{3

WolframAlpha

Plots:



(*u* from −2.5 to 0)



(*u* from −4 to 2)

y = \frac{2e^{−|u+1.299|}}{3} −\frac{e^{−2|u+1.299|}}{3

WolframAlpha