Traffic Flow Modeling and Car Accident Risk

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Background

- Traffic modeling uses PDEs to simulate traffic flow
- Used to predict and prevent traffic jams
- Most models assume car crashes cannot happen
- However, predicting accidents is also very important
- Crashes can be permitted using a coupled model
- Combination of a classic traffic model and gas flow model
The Aw-Rascle Model

- Second-order model: velocity \((v)\), density \((\rho)\)
- Anticipation factor \((p)\) analogous to pressure
- First equation conserves mass
- Second equation conserves momentum
- Has a maximum density, no collisions possible

\[
\begin{align*}
\partial_t \rho + \partial_x (\rho v) &= 0, \\
\partial_t (\rho w) + \partial_x (\rho w v) &= 0, \\
w &= v + p(\rho).
\end{align*}
\]
The Pressureless Gas Dynamics Model

- Comes from gas flow modeling
- Very similar structure to AR model
- $w = v + p(\rho)$ replaced by $v$
- Since drivers do not anticipate traffic, crashes can occur
- Delta shocks, where density increases without bound

\[
\begin{align*}
\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2) &= 0, \\
\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v) &= 0,
\end{align*}
\]
Numerical Approximation Methods

- McCormack Scheme
- \( U = (\rho, \rho w)^T, F = vU \)

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\end{align*}
\]

Predictor

\[
\tilde{U}_i^{n+1} = U_i^n - \frac{dt}{dx} (F_i^n - F_{i-1}^n)
\]  

Corrector

\[
U_i^{n+1} = 0.5(U_i^n + \tilde{U}_i^{n+1}) - 0.5 \frac{dt}{dx} (\tilde{F}_{i+1}^{n+1} - \tilde{F}_i^{n+1})
\]

- Requires a smoothing step to reduce spurious oscillation
Numerical Approximation Methods

- Godunov-Type Scheme
- \( Q = (\rho, \rho v)^T, F = vU \)
- Uses values at interfaces

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} &= 0, \\
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho v^2)}{\partial x} &= 0,
\end{align*}
\]

\[
Q_{i}^{n+1} = Q_{i}^{n} - \frac{\Delta t}{\Delta x} (F_{i+1/2} - F_{i-1/2}) - \frac{\Delta t}{\Delta x} (\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2})
\]

- Correction term gives second order accuracy
Numerical Approximation Methods

My model, using McCormack and Godunov schemes
Graphs position vs. density at t=0 and t=150

A model using Godunov schemes only
Graphs position vs. time, density as color
Same situation as before, but with lower velocity in the PGD zone - an accident occurs.
Graphs of position vs density are at t=0 and t=65.
References

- Modeling road traffic accidents using macroscopic second-order models of traffic flow
- The Dynamics of Pressureless Dust Clouds and Delta Waves
- Improved Numerical Method for Aw-Rascle Type Continuum Traffic Flow Models