Exercise C1. What is the centroid of a solid hemisphere?

- spherical coordinates
- volume = 0.5 * volume of a sphere
- by symmetry, the x,y coordinates of the centroid should be 0
- phi bar = 0, thetabar = 0
- only have to solve for rho bar
- theta from 0 to 2pi
- phi from 0 to pi/2
- rho from 0 to R

- with these bounds, integrate rho * dV
- really nice integral

Exercise C4. Compute the volume of an ellipsoid (3d version of an ellipse) with semiaxes \(a, b, c\) by using a linear change of variables.

\[
\text{matrix is } \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \\
\text{det} = abc \\
\text{volume of ellipsoid} = abc \ast \text{volume of a unit sphere}
\]
Exercise C5. Compute the integral

\[\int \int x^2 y^2 (y^2 - x^2) \, dx \, dy\]

over the domain \(1 \leq xy \leq 4\) and \(1 \leq y - x \leq 3\).

Exercise C6. Evaluate

\[\int \int xy (x^2 + y^2) \, dx \, dy\]

over the domain given by \(1 \leq xy \leq 4\) and \(1 \leq x^2 - y^2 \leq 3\).
Exercise C2. What is the moment of inertia of a unit-density solid ball of radius $R$ about the centroid?