
Math 202 - Bonus Problem 1

Due on November 4 (with your regular homework).

This problem is entirely optional, and will not hurt your grade at all (if you don't even try to solve it).

Given a sequence $\{a_n\}_{n \geq 1}$ of real numbers, define

$$b_n = \frac{1}{n} \sum_{k=1}^n a_k.$$

This is the “Cesàro Sum” of a_n .

Part 1. Suppose that $b_n \rightarrow L$ as $n \rightarrow \infty$ with $L \in \mathbb{R}$. Does $a_n \rightarrow L$ as $n \rightarrow \infty$? If so, then prove it. If not, then provide a counter example.

Part 2. Suppose that $|b_n| \leq M < \infty$ for all $n \in \mathbb{N}$. Does this imply that $\{a_n\}_{n \geq 1}$ is bounded? If so, then prove it. If not, then provide a counter example.