Name: ______________________________

(Practice) In Class MidTerm Exam for Math 113

10 - 11:30 am, March 20, 2008

Problem	Points	Score
1	??	
2	??	
3	??	
4	??	
5	??	
6	??	
7	??	
Total	100	

- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- Some problems will be asking you to prove results covered in class. For those that do not, you may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators are permitted.
- **This is a practice exam.**

1. (a) Give the definition of a conformal map.

   (b) Give the condition(s) for a holomorphic map to be conformal.

   (c) Consider the wedge $\Omega = \{z \in \mathbb{C} \,\big|\, 0 < arg\{z - 3i\} < \frac{\pi}{3}\}$. Give a conformal mapping of Ω into the unit disk $B_1(0) = \{z \in \mathbb{C} \,\big|\, |z| < 1\}$ and justify your answer.

2. Suppose that $a, b \in \mathbb{C}$ are two vertices of a square. Find the two other vertices in all possible cases.

3. Let A be the complement in $\mathbb{C}$ of the non-positive real axis, i.e. $A = \mathbb{C} - \{x \in \mathbb{R} \mid x \leq 0\}$.

   (a) Show that there exists a unique function f, which is holomorphic on A and satisfies $f(x) = x^x$ for all real $x > 0$.

   (b) Find $f(i)$ and $f'(i)$.

4. (a) Prove or disprove: There exists a holomorphic mapping from $\mathbb{C}$ to the upper half plane

$$\{z \in \mathbb{C} \,\big|\, Im\{z\} > 0\}.$$

(b) Prove or disprove: The image of $\mathbb{C}$ under a nonconstant entire mapping is dense in $\mathbb{C}$.

5. Consider the stereographic projection of the sphere of radius 1 centered at the origin in $\mathbb{R}^3$

$$\{ (x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$$

from the south pole $(0, 0, -1)$ onto the xy-plane. Derive the expression for the image of (x_1, x_2, x_3) under this map and write it out as a complex number.

6. Let f be entire and $u(x, y) = \text{Re}(f(x + iy))$. Suppose that

$$u(x, y) = u(-y, x).$$

Prove that for all $z \in \mathbb{C}$, $f(z) = f(iz)$.

7. Show that if F is analytic on A, then so is f where

$$f(z) = \frac{F(z) - F(z_0)}{z - z_0},$$

if $z \neq z_0$ and $f(z_0) = F'(z_0)$ where z_0 is some point of A.

Extra space for work: