Mathematics 112. Real Analysis, Spring 2006

Syllabus

Instructor: Robert Strain

Class schedule: TH 10-11:30 in SC 507, attendance is *manditory.*

Required Text: Rudin, *Principles of Mathematical Analysis, 3rd Ed.*

Advanced Text (Not Required): Lieb & Loss, *Analysis, 2nd Ed.* (For Lebesgue Theory & further reading.)

Prerequisites: Some knowledge of formal proofs will be helpful, but not required. Your ability to write proofs will develop in this class. For more about learning to write mathematical proofs, see:

Solow, *How to read and do proofs, 4th Ed.* **(Not Required)**

Reserve: All three of these texts are on reserve at Cabot.

Course description: An introduction to mathematical analysis and the theory behind calculus. An emphasis on learning to understand and construct proofs. Specific topics covered are listed on the back.

Homework: Weekly homework assignments will be due on Thursday. Collaboration between students is encouraged, but you must write your own solutions, understand them and give credit to your collaborators.

Late homework will not be accepted.

Reading and Lectures: Please read the book before class! Students are responsible for all topics covered in the readings and lectures. Lectures may go beyond the reading, and not every topic in the reading will be covered in class.

Exams: *In-class* midterm on Thursday, March 23 (*no makeups*).

Take-home final handed out Tuesday, May 9 and due Thursday, May 11.

Grading: Your final grade will be based on homework and quizzes (40%), the midterm (20%) and the final (40%).

Course Webpage: `www.math.harvard.edu/~strain/ma112/`

Topics to be covered (subject to change):

- The set of real numbers: ordered sets; fields; Euclidean spaces.
- Basic Topology and metric spaces: countable and uncountable sets; metric spaces; open and closed sets; compactness and Weierstrass theorem; connectedness.
- Convergence: Cauchy sequences; subsequences; upper and lower limits; completeness of real numbers; convergence and absolute convergence of series.
- Continuity: Limits; continuous functions on metric spaces; continuity and compactness/connectedness; intermediate value theorem; discontinuities; monotonic functions.
- Differentiation: Left and right derivatives; mean value theorem, differentiability; l'hospital's rule; Taylor theorem.
- Riemann-Stieltjes integral: existence of Riemann integrals; change of variable; proof of the fundamental theorem of calculus; integraion by parts.
- Sequences and Series of functions: Uniform convergence of functions; Equicontinuity and Arzela-Ascoli theorem; Stone-Weierstrass theorem.
- Special functions: the exponential and logarithmic functions; trigonometric functions; fourier series; the gamma function.
- Functions of several variables; the contraction principle; the inverse function theorem and the implicit function theorem.
- The Lebesgue theory: set functions; outer measures; construc- tion of the Lebesgue measure; measurable sets and measurable functions; integration; Lebesgue's monotone convergence theorem; Fatou's lemma; Lebesgue's dominated convergence theorem; comparison between the Lebesgue integral and the riemann integral; L^2 spaces.