2.3.T/F.2

False. These planes might have a line in common.

2.3.T/F.4

True. If $\vec{x}$ and $\vec{y}$ are two solutions, then any $a\vec{x} + (1-a)\vec{y}$ will be a solution. In other words, all points on the line passing through $\vec{x}$ and $\vec{y}$ are solutions.

2.3.T/F.6

False. A counter example $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

2.3.Prob.12

$$\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t),$$

where

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} -4 & 3 \\ 6 & -4 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} 4t \\ t^2 \end{bmatrix}.$$

2.3.Pro.16

$$\mathbf{x}' = \begin{bmatrix} 4e^{4t} \\ -8e^{4t} \end{bmatrix},$$

and

$$A\mathbf{x} + \mathbf{b} = A\mathbf{x} = \begin{bmatrix} 2e^{4t} + 2e^{4t} \\ -2e^{4t} - 6e^{4t} \end{bmatrix} = \begin{bmatrix} 4e^{4t} \\ -8e^{4t} \end{bmatrix}.$$

Hence

$$\mathbf{x}' = A\mathbf{x} + \mathbf{b}.$$

2.4.T/F.2

False. It doesn't necessarily have leading ones.

2.4.T/F.4

False. The number of rows doesn't decide the rank.

2.4.T/F.8

True.

2.4.Prob.6

It is in REF but not RREF.

2.4.Prob.12

We start from $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 4 \\ 0 & 3 & 5 \end{bmatrix}$. First do $A_{12}(-1)$ and $A_{13}(-3)$ and we get $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$. Another $A_{23}(4)$ leads to $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. The rank is 2.

2.5.T/F.2

True. 0 is always a solution.

2.5.T/F.4

True.

2.5.Prob.16

We do a series of row operation on $[A\,\mathbf{b}]$ as follows.

$$\begin{bmatrix} 1 & -3 & 1 & 8 \\ 5 & -4 & 1 & 15 \\ 2 & 4 & -3 & -4 \end{bmatrix} \overset{A_{12}(-5),A_{13}(-2)}{\longrightarrow} \begin{bmatrix} 1 & -3 & 1 & 8 \\ 0 & 11 & -4 & -25 \\ 0 & 10 & -5 & -20 \end{bmatrix} \overset{M_2(1/11)}{\longrightarrow} \begin{bmatrix} 1 & -3 & 1 & 8 \\ 0 & 1 & -4/11 & -25/11 \\ 0 & 10 & -5 & -20 \end{bmatrix}$$

$$\overset{A_{23}(-10)}{\longrightarrow} \begin{bmatrix} 1 & -3 & 1 & 8 \\ 0 & 1 & -4/11 & -25/11 \\ 0 & 0 & -15/11 & 30/11 \end{bmatrix} \overset{M_3(-11/15)}{\longrightarrow} \begin{bmatrix} 1 & -3 & 1 & 8 \\ 0 & 1 & -4/11 & -25/11 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

$$\overset{A_{31}(-1),A_{32}(4/11)}{\longrightarrow} \begin{bmatrix} 1 & -3 & 0 & 10 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \overset{A_{21}(3)}{\longrightarrow} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Therefore $\mathbf{x} = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$.

2.5.Prob.48

We do a series of row operation on $[A]$ as follows.

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ -1 & 0 & -1 & 2 \\ 1 & 3 & 2 & 2 \end{bmatrix} \overset{A_{12}(1),A_{13}(-1)}{\longrightarrow} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix} \overset{A_{23}(-2)}{\longrightarrow} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \overset{A_{31}(-1),A_{21}(-1)}{\longrightarrow} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Let $x_4 = t$ then $x_2 = x_3 = -x_4 = -t$ and $x_1 = 3x_4 = 3t$. Therefore the solutions to the system is

$$\left\{ (3t, -t, -t, t) \,\middle|\, t \in \mathbb{C} \right\},$$

or replace $\mathbb{C}$ by any field under consideration.

2.6.T/F.2

False. For example $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

2.6.T/F.4

False. A and B can both be non square matrices.

2.6.T/F.6

True. $(AB)^{-1} = B^{-1}A^{-1}$.

2.6.Prob.12

We do a series of row operation on $[A\, I_3]$ as follows.

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 6 & -2 & 0 & 1 & 0 \\ -1 & 1 & 4 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{12}(-2), A_{13}(1)} \begin{bmatrix} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 2 & 4 & -2 & 1 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{M_2(1/2)} \begin{bmatrix} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1/2 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 \end{bmatrix}$$
$$\xrightarrow{A_{23}(-3)} \begin{bmatrix} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1/2 & 0 \\ 0 & 0 & -5 & 4 & -3/2 & 1 \end{bmatrix} \xrightarrow{M_3(-1/5)} \begin{bmatrix} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -4/5 & 3/10 & -1/5 \end{bmatrix}$$
$$\xrightarrow{A_{31}(3), A_{32}(-2)} \begin{bmatrix} 1 & 2 & 0 & -7/5 & 9/10 & -3/5 \\ 0 & 1 & 0 & 3/5 & -1/10 & 2/5 \\ 0 & 0 & 1 & -4/5 & 3/10 & -1/5 \end{bmatrix} \xrightarrow{A_{21}(-2)} \begin{bmatrix} 1 & 0 & 0 & -13/5 & 11/10 & -7/5 \\ 0 & 1 & 0 & 3/5 & -1/10 & 2/5 \\ 0 & 0 & 1 & -4/5 & 3/10 & -1/5 \end{bmatrix}.$$

Therefore $A^{-1} = \begin{bmatrix} -13/5 & 11/10 & -7/5 \\ 3/5 & -1/10 & 2/5 \\ -4/5 & 3/10 & -1/5 \end{bmatrix}$.

2.6.Prob.30

If $A^T = -A$, then $(A^{-1})^T = (A^T)^{-1} = (-A)^{-1} = -A^{-1}$, hence A^{-1} is skew-symmetric.

2.6.Prob.34

If $\Delta \neq 0$,

$$\frac{1}{\Delta} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} A = \frac{1}{\Delta} \begin{bmatrix} a_{22}a_{11} - a_{12}a_{21} & a_{22}a_{12} - a_{12}a_{22} \\ -a_{21}a_{11} + a_{11}a_{21} & -a_{21}a_{12} + a_{11}a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Therefore

$$A^{-1} = \frac{1}{\Delta} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}.$$