Math 361- Quiz 7

April 10, 2014

1. (Worth 10) Recall Stokes' Theorem: If Ψ is a k-chain of class C'' in open set $V \subset \mathbb{R}^m$ and $\omega \in \Omega^{k-1}(V)$ is of C' class, then

$$\int_{\Psi} d\omega = \int_{\partial\Psi} \omega \tag{1}$$

For this problem we will consider $k = 1$ and $m = 1$ and Ψ will be an affine oriented 1-simplex, i.e.

$$\Psi = [\mathbf{p}_0, \mathbf{p}_1] = [a, b] \qquad \text{where } a, b \text{ are scalars.}$$

(a) (Worth 3) Write down $\Psi(u)$ in the form of $\mathbf{p}_0 + Au$ as was done in class. What is the parameter space D for Ψ?

Proof. We have that $D = Q^1 = [0, 1]$ and

$$\Psi(u) = a + (b - a)u$$

□

(b) (Worth 5) Recall the definition of integrating forms and boundary of a simplex. Compute the LHS and RHS of (1) in this special case.

Proof. We first compute the LHS. Note that ω is a zero-form, so we can write $\omega = f(x)$ where $x \in \mathbb{R}$. Then $d\omega = f'(x)dx$. Hence the LHS becomes

$$\int_{\Psi} d\omega = \int_{\Psi} f'(x)dx$$

Note that we still have an integral of a 1 form!!! Thus, we have to use the definition of integration of a 1-form. Hence, we have

$$\int_{\Psi} f'(x)dx = \int_{D} f'(\Psi(u))\frac{dx}{du}du = \int_{0}^{1} f'(a + (b - a)u)(b - a)du = \int_{a}^{b} f'(x)dx$$

Note now that the integral in the last equality is the integral of a function, which can easily be done from Calculus 1. We now compute the RHS. Note that $\partial\Psi = [b] - [a]$. Hence,

$$\int_{\partial\Psi} \omega = \int_{[b]-[a]} f(x) = \int_{[b]} f(x) + \int_{-[a]} f(x) = f(b) - f(a)$$

Note that last equality is by definition of integrating a zero form. □

(c) (Worth 1) Do we have equality of the LHS and RHS? What does this prove, i.e. what is the name of this theorem? DO NOT JUST SAY A SPECIAL CASE OF STOKES!

Proof. We do have equality of the LHS and the RHS

$$\int_a^b f'(x)dx = f(b) - f(a)$$

This is the Fundamental Theorem of Calculus! □

(d) (Worth 1) Do we need ω to be of C' class for this special case of Stokes?

Proof. For the Fundamental Theorem of Calculus you do not need f to be C'. This is because, the general form is

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$. □