Math 240: Triple Integrals

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Thursday February 2, 2012

Outline

Today's Goals

1. Be able to set up and evaluate triple integrals in cartesian coordinates.
2. Be able to set up and evaluate triple integrals in cylindrical coordinates.
3. Be able to set up and evaluate triple integrals in spherical coordinates

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$$\int\int\int F(x,y,z)dV = lim_{|P|\to 0}\Sigma_{k=1}^{n} F(x_k^*, y_k^*, z_k^*)V_k$$

Evaluating a Triple Integral

To evaluate a triple integral you must cut the region you are integrating over into pieces of the form
$a \leq x \leq b$ and $f_1(x) \leq y \leq f_2(x)$ and $h_1(x, y) \leq z \leq h_2(x, y)$.

$$\int\int\int F(x, y, z)dV = \int_a^b \int_{f_1(x)}^{f_2(x)} \int_{h_1(x,y)}^{h_2(x,y)} F(x, y, z)dzdydx$$

Evaluating a Triple Integral in Cylindrical

To evaluate a triple integral you must cut the region you are integrating over into pieces of the form
$a \leq \theta \leq b$ and $f_1(\theta) \leq r \leq f_2(\theta)$ and $h_1(r,\theta) \leq z \leq h_2(r,\theta)$.

$$\int\int\int F(x,y,z)dV = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} \int_{h_1(r,\theta)}^{h_2(r,\theta)} F(x,y,z)rdzdrd\theta$$