MATH 240 - Spring 2012
Practice Midterm Two

Name:

TA:

Recitation Time:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	**Score (out of)**
1	
2	
3	
4	
5	
Total	

1. Find the general solution to the following D.E.

$$y''' + 3y'' + 3y' + y = 0$$

aux. eq $\quad m^3 + 3m^2 + 3m + 1 = 0$

$(m+1)^3 = 0$

$m = -1$ (mult. 3)

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ \rightarrow 1 & & 3 & & 3 & & 1 \end{array}$$

$$\boxed{y_g = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}}$$

2. For what value of k does the following system have ∞-many solutions?

$$\begin{pmatrix} k & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 2 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

∘ $A v = \vec{0}$ has ∞-many sol. iff $\det(A) = 0$.

$$\det\begin{pmatrix} k & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 2 & k \end{pmatrix} = -0 \begin{vmatrix} 2 & 1 \\ 2 & k \end{vmatrix} + 1 \begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} + 2 \begin{vmatrix} k & 2 \\ 1 & 2 \end{vmatrix}$$

$$= k^2 - 1 + 2(2k - 2)$$

$$= k^2 + 4k - 5$$

$$k^2 + 4k - 5 = 0$$

$$(k-1)(k+5) = 0$$

$$\boxed{k = 1, -5}$$

3. Given the matrix A find the diagonal matrix D and the invertible matrix P such that $P^{-1}AP = D$

$$A = \begin{pmatrix} -9 & 13 \\ -2 & 6 \end{pmatrix}$$

Step 1: Find e. val. s

$$\begin{vmatrix} -9-\lambda & 13 \\ -2 & 6-\lambda \end{vmatrix} = 0$$

$$(\lambda+9)(\lambda-6)+26=0$$

$$\lambda^2+3\lambda-54+26=0$$

$$\lambda^2+3\lambda-28=0$$

$$(\lambda-4)(\lambda+7)=0$$

$\lambda = 4$ or -7

Find e. vec.

$\lambda = 4$

$$\begin{bmatrix} -13 & 13 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x+2y=0$$

$$x=y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ x \end{bmatrix} = x\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Find e. vec. for $\lambda = -7$

$$\begin{bmatrix} -2 & 13 \\ -2 & 13 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-2x+13y=0$$

$$x = \frac{13}{2}y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{13}{2} \\ 1 \end{bmatrix} y =$$

$$\boxed{P = \begin{bmatrix} 1 & \frac{13}{2} \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 0 \\ 0 & -7 \end{bmatrix}}$$

4. Find a 3×3 matrix A with the following properties

- A has eigenvalue 1 with eigenvector $< 1, 1, 0 >$.
- A has eigenvalue 2 with eigenvector $< 0, 1, 1 >$.
- A has eigenvalue 3 with eigenvector $< 2, 0, 1 >$.

• A has 3 distinct e. val. so A is diagonalizable.

So $A = PDP^{-1}$ where $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Find P^{-1}:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 - R_1 \to R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - R_2 \to R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 3 & 1 & -1 & 1 \end{array}\right]$$

$$\tfrac{1}{3} R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/3 & -1/3 & 1/3 \end{array}\right]$$

$$\begin{array}{l} R_2 + 2R_3 \to R_2 \\ R_1 - 2R_3 \to R_1 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 2/3 & -2/3 \\ 0 & 1 & 0 & -1/3 & 1/3 & 2/3 \\ 0 & 0 & 1 & 1/3 & -1/3 & 1/3 \end{array}\right] \quad P^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ -1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ -1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} 1 & 0 & 6 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ -1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 7 & -4 & 4 \\ 1 & 4 & 2 \\ 1 & -1 & 7 \end{bmatrix} = \begin{bmatrix} 7/3 & -4/3 & 4/3 \\ 1/3 & 4/3 & 2/3 \\ 1/3 & -1/3 & 7/3 \end{bmatrix}$$

5. If A is a square matrix such that $A^2 + 2A = 3I$, what are the possible eigenvalues of A? Justify your answer.

Let v be an eigen vector of A.

$(A^2 + 2A)v = (3I)v$

$A^2 v + 2Av = 3v$

$A(\lambda v) + 2\lambda v = 3v$

$\lambda^2 v + 2\lambda v = 3v$

$\lambda^2 v + 2\lambda v - 3v = \vec{0}$

$(\lambda^2 + 2\lambda - 3)v = \vec{0}$

Since $\vec{v} \neq \vec{0}$, $\lambda^2 + 2\lambda - 3 = 0$

$(\lambda + 3)(\lambda - 1) = 0$

$\boxed{\lambda = -3 \text{ or } 1}$