MATH 240 - Spring 2012
Practice Midterm One

Name:

TA:

Recitation number:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	**Score (out of)**
1	
2	
3	
4	
5	
Total	

1. If $\mathbf{F} = < xy, y^2z, z^3 >$, evaluate $\int\int_S (\mathbf{F} \cdot \mathbf{n}) dS$ where S is the outward oriented boundary of the region bounded by $z = 0$, $z = 1$, $y = 1 - x$, $x = 0$ and $y = 0$.

2. Derive Green's Theorem using Stokes' Theorem.

3. Let C_1 and C_2 be the closed curves

$$C_1 = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}, \quad C_2 = \{(x,y) \in \mathbb{R}^2 \mid 4x^2 + 9y^2 = 36\}$$

on the (x,y)-plane, oriented counterclockwise. Consider the line integrals

$$\oint_{C_i} \frac{(x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y}{x^2+y^2}, \quad i = 1, 2\,.$$

(a) Are the two integrals $\displaystyle\oint_{C_1} \frac{(x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y}{x^2+y^2}$ and $\displaystyle\oint_{C_2} \frac{(x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y}{x^2+y^2}$ equal? Why? (**Justify your answer**.)

(b) Evaluate these two line integrals.

$$\oint_{C_1} \frac{(x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y}{x^2+y^2} = \underline{\qquad\qquad\qquad\qquad}$$

$$\oint_{C_2} \frac{(x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y}{x^2+y^2} = \underline{\qquad\qquad\qquad\qquad}$$

4.

a) Find a primitive of the differential $e^{2z}dx + 3y^2dy + 2xe^{2z}dz$.

b) Use the answer from part a) to evaluate the following line integral

$$\int_{(1,1,ln(3))}^{(2,2,ln(3))} e^{2z}dx + 3y^2dy + 2xe^{2z}dz$$

5. Let S be the portion of the outward oriented sphere $x^2+y^2+(z-1)^2=4$ above the plane $z=0$. Evaluate $\int\int_S (curl(\mathbf{F})\cdot\mathbf{n})dS$ when $\mathbf{F}=<xy^2+y+e^{z^2}, x^2y+ze^z, xyz>$.