Math 240: Solving Systems of DEs by Diagonalization

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Outline

1. Review

2. Today's Goals

3. Diagonalization and Systems

4. Review of Power Series

Review of Last Time

1. Used phase portraits to make qualitative and quantitative statements about systems.

Solutions to 2 by 2 systems

1. Two linear solutions in the phase plane implies
2. One linear solution in the phase plane implies
3. No linear solutions implies

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2. One linear solution in the phase plane implies one repeated real eigenvalue.
3. No linear solutions implies complex eigenvalues.

Today's Goals

1. Use diagonalization to solve systems of linear DEs.
2. Review power series.

Coupled and Uncoupled Systems

Definition

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A system $\mathbf{X}' = \mathbf{AX}$ is **uncoupled** if each equation is of the form $x_i' = c_i x_i$ where c_i is some constant.

If $\mathbf{A}$ is diagnolizable, then the system $\mathbf{X}' = \mathbf{AX}$ can be uncoupled.

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Given a system $\mathbf{X}' = \mathbf{AX}$. Suppose $\mathbf{A}$ is diagnolizable(i.e. $P^{-1}AP = D$).
If $\mathbf{X} = \mathbf{PY}$ is a solution to $\mathbf{X}' = \mathbf{AX}$, then

$$\mathbf{Y} = \begin{bmatrix} c_1 e^{\lambda_1 t} \\ \vdots \\ c_n e^{\lambda_n t} \end{bmatrix}$$

Review of Power Series

Definition

$$\sum_{n=o}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

is a **power series centered at a**.

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Definition

The **radius of convergence** is the largest R such that $\sum_{n=o}^{\infty} c_n(x-a)^n$ converges for all x such that $|x-a| < R$.

Finding the Radius of Convergence

Ratio Test

Let

$$lim_{n\to\infty}|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}| = |x-a|lim_{n\to\infty}|\frac{c_{n+1}}{c_n}| = L$$

If $L < 1$ the series converges. If $L > 1$ the series diverges. If $L = 1$ we don't know.

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Find the radius of convergence for $\sum_{n=o}^{\infty} \frac{(x)^n}{n!}$