Math 240: Phase Portraits

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Outline

1. Review
2. Today's Goals
3. Distinct Eigenvalues

Review of Last Time

1. Learned how to solve constant coefficient systems.

Solutions to 2 by 2 systems

General Solution 2 by 2 System with Distinct Real Eigenvalues λ_1 and λ_2:

$$\mathbf{X} = c_1\mathbf{K}_1 e^{\lambda_1 t} + c_2\mathbf{K}_2 e^{\lambda_2 t}$$

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$$c_1\mathbf{K}e^{\lambda t} + c_2[\mathbf{K}te^{\lambda t} + \mathbf{P}e^{\lambda t}]$$

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General Solution 2 by 2 System with Complex Eigenvalue $\alpha + i\beta$:

$$c_1[Re(\mathbf{K})cos(\beta t) - Im(\mathbf{K})sin(\beta t)]e^{\alpha t}$$

$$+c_2[Im(\mathbf{K})cos(\beta t) + Re(\mathbf{K})sin(\beta t)]e^{\alpha t}$$

Today's Goals

1. Use phase portrait to develop intuition for systems.

Guessing a Solution

A solution to a 2 by 2 initial value system gives a curve $r(t) =< x(t), y(t) >$ in the xy-plane (the **Phase Plane**).

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We can gain qualitative and quantitative information about a system by looking at its Phase Portrait.
We will use pplane, available here:
http://math.rice.edu/∼dfield/dfpp.html