Even More Power Series Solutions to D.E.s at Regular Singular Points

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Monday April 23, 2012

Outline

Final exam info

1. Friday May 4th from noon to 2pm in Stitler Hall
2. One 8.5" by 11" page of notes allowed.
3. Arrive 5 to 10 min early
4. Must bring student ID.

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Help and study materials

1. My office hours this week: Mon 2-3, Wed 10:30-11:30, Fri 10:30-11:30
2. My office hours next week: Mon 10:30-11:30, Wed 10:30-11:30, Thurs 5-7
3. Tomorrow Recitation
4. Practice final and solutions posted later this week
5. Old Practice finals, and old finals

The Frobenius method for find solutions at regular singular points

To solve $y'' + P(x)y' + Q(x)y = 0$ at a regular singular point x_0, substitute

$$y = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}$$

and solve for r and the c_n to find a series solution centered at x_0.

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and solve for r and the c_n to find a series solution centered at x_0.
We may not find two linearly independent solutions this way!

Today's Goals

1. Deal with exceptional cases of finding power series solutions to D.E.s at regular singular points.

Indicial Roots

To find the r in $y = \sum_{n=0}^{\infty} c_n(x - x_0)^{n+r}$ we substitute the series into $y'' + P(x)y' + Q(x)y = 0$ and equate the total coefficient of the lowest power of x to zero. This will be a quadratic equation in r.

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The roots, r_1 and r_2, we get are the **indicial roots** of $y'' + P(x)y' + Q(x)y = 0$

Cases

Case 1: If r_1 and r_2 are distinct and do not differ by an integer, then we get two linearly independent solutions

$$y_1 = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r_1} \quad and \quad y_2 = \sum_{n=0}^{\infty} b_n (x - x_0)^{n+r_2}$$

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Case 2: In all other cases we get two linearly independent solutions of the form

$$y_1 = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r_1} \ \ and \ \ y_2 = C y_1(x) ln(x) + \sum_{n=0}^{\infty} b_n (x - x_0)^{n+r_2}$$