Math 240: Power Series Solutions to D.E.s

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Outline

Today's Goals

1. Find power series solutions to D.E.

Review of Power Series

Definition

$$\sum_{n=o}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$

is a **power series centered at a**.

Definition

A function f is **analytic** at a point a if it can be represented by a power series in $x - a$ with a positive radius of convergence.

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Given a differential equation $y'' + P(x)y' + Q(x)y = 0$, a point x_0 is an **ordinary point** if both $P(x)$ and $Q(x)$ are analytic at x_0. If a point in not ordinary it is a **singular point**.

Theorem

If x_0 is an ordinary point of $y'' + P(x)y' + Q(x)y = 0$, there are always two linearly independent power series solutions centered at x_0 and each has a radius of convergence at least the distance from x_0 to the closest singular point.

Solving D.E.s Using Power Series

Given the differential equation $y'' + P(x)y' + Q(x)y = 0$, substitute

$$y = \sum_{n}^{\infty} c_n (x-a)^n$$

and solve for the c_n to find a power series solution centered at a.