Math 240: Sequences, Series and Power Series

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Outline

Review of Sequences

Definition

A sequence is a infinite collection of numbers, one for each natural number, and is denoted by $\{a_n\}_{n=0}^{\infty}$.

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If $\{a_n\}_{n=0}^{\infty}$ converges to L we say $lim_{n\to\infty} a_n = L$.

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Review of Power Series

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$$\sum_{n=o}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$

is a **power series centered at a**.

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A Taylor series always approximates a function at the value a, but how well?

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What is the radius of convergence for the Taylor series for e^x at $x = 0$?

Finding the Radius of Convergence

Ratio Test

Let

$$lim_{n\to\infty}|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}| = |x-a|lim_{n\to\infty}|\frac{c_{n+1}}{c_n}| = L$$

If $L < 1$ the series converges. If $L > 1$ the series diverges. If $L = 1$ we don't know.

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Find the radius of convergence for $\sum_{n=o}^{\infty} \frac{(x)^n}{n!}$

Necessary Algebra Skills

1. Shifting the summation index
2. Adding two power series