Math 240: Diagonalization

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Outline

Today's Goals

1. Be able to diagonalize matrices.
2. Be able to use diagonalization to compute high powers of matrices.

Important Examples

1. A matrix may have no eigenvaules (We don't count non-real eigenvalues)
2. A matrix may have multiple eigenvectors for a single eigenvalue.
3. A $n \times n$ matrix may not have n linearly independent eigenvectors.

Diagonalizability

Definition

An $n \times n$ matrix A is **diagonalizable** if there exists an $n \times n$ invertible matrix P and an $n \times n$ diagonal matrix D such that $P^{-1}AP = D$.

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Example: Find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

Diagonalizability Theorems

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A $n \times n$ matrix is diagonalizable if and only if it has n linearly independent eigenvectors.

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Note: Not all diagonalizable matrices have n distinct eigenvalues.

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Example: Given

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

compute A^8.