Math 240: Stoke's Theorem

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Outline

What to Study for the Midterm

1. Practice Midterm (Posted this weekend)
2. My Old Practice Midterm (http://www.math.upenn.edu/∼ryblair/Math240/papers/PracMT2.pdf)
3. My Old Practice Final (http://www.math.upenn.edu/∼ryblair/Math240/papers/PracFinal.pdf)
4. Other Old Finals (http://www.math.upenn.edu/ugrad/calc/m240/oldexams.html)
5. Homework, Quizzes and Examples in Class

Stokes' Theorem

Theorem

Let S be an **nice** *oriented surface bounded by a* **nice** *curve C. Let $F = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ be a* **nice** *vector field. If C is traversed in the positive direction and $\mathbf{T}$ is the unit tangent vector to C then*

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T})ds = \int\int_S (curl(F) \circ \mathbf{n})dS$$

where $\mathbf{n}$ is the unit normal to S in the direction of the orientation of S.

Example Use Stokes' theorem to evaluate $\oint_C F \circ d\mathbf{r}$ where C is the intersection of $x^2 + y^2 = 1$ and $x + y + z = 1$ oriented counter clockwise from above and $F = y^3\mathbf{i} - x^3\mathbf{j} + z^3\mathbf{k}$.

Example: Use Stokes' theorem to evaluate $\oint_C F \circ d\mathbf{r}$ where C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ oriented counterclockwise when viewed from above and $F = (2z + x)\mathbf{i} + (y - z)\mathbf{j} + (x + y)\mathbf{k}$.

Example: Let S be the portion of $z = x^2 + 4y^2$ lying beneath the plane $z = 1$. Orient S upward. Find the flux of curl(F) across S for $F = y\mathbf{i} - xz\mathbf{j} + xz^2\mathbf{k}$.

Stokes' Theorem

Theorem

Let S be a piecewise smooth oriented surface bounded by a piecewise smooth simple closed curve C. Let $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ *be a vector field for which P, Q and R are continuous and have continuous partial derivatives in the region of 3-space containing S. If C is traversed in the positive direction and* $\mathbf{T}$ *is the unit tangent vector to C then*

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T})ds = \int\int_S (curl(F) \circ \mathbf{n})dS$$

where $\mathbf{n}$ *is the unit normal to S in the direction of the orientation of S.*