# Math 240: Stoke's Theorem

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Friday February 3, 2012

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Friday February 3, 2012 1 / 8

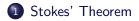
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- Practice Midterm (Posted this weekend)
- My Old Practice Midterm (http://www.math.upenn.edu/~ryblair/Math240/papers/PracMT2.pdf)
- My Old Practice Final (http://www.math.upenn.edu/~ryblair/Math240/papers/PracFinal.pdf)
- Other Old Finals (http://www.math.upenn.edu/ugrad/calc/m240/oldexams.html)
- Homework, Quizzes and Examples in Class

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# Stokes' Theorem

#### Theorem

Let *S* be an **nice** oriented surface bounded by a **nice** curve *C*. Let  $F = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$  be a **nice** vector field. If *C* is traversed in the positive direction and **T** is the unit tangent vector to *C* then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

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**Example** Use Stokes' theorem to evaluate  $\oint_C F \circ d\mathbf{r}$  where *C* is the intersection of  $x^2 + y^2 = 1$  and x + y + z = 1 oriented counter clockwise from above and  $F = y^3 \mathbf{i} - x^3 \mathbf{j} + z^3 \mathbf{k}$ .

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**Example:** Use Stokes' theorem to evaluate  $\oint_C F \circ d\mathbf{r}$  where *C* is the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) oriented counterclockwise when viewed from above and  $F = (2z + x)\mathbf{i} + (y - z)\mathbf{j} + (x + y)\mathbf{k}$ .

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**Example:** Let *S* be the portion of  $z = x^2 + 4y^2$  lying beneath the plane z = 1. Orient *S* upward. Find the flux of curl(F) across *S* for  $F = y\mathbf{i} - xz\mathbf{j} + xz^2\mathbf{k}$ .

3

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# Stokes' Theorem

### Theorem

Let S be a piecewise smooth oriented surface bounded by a piecewise smooth simple closed curve C. Let  $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$  be a vector field for which P, Q and R are continuous and have continuous partial derivatives in the region of 3-space containing S. If C is traversed in the positive direction and **T** is the unit tangent vector to C then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

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