Math 240: Eigenvalues and Eigenvectors

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Outline

Today's Goals

1. Review Eigenvectors and Eigenvalues.
2. Be able to diagonalize matrices.
3. Be able to use diagonalization to compute high powers of matrices.

Review of Eigenvalues and Eigenvectors

Definition

Let λ be a scalar, x be a $n \times 1$ column vector and A be a $n \times n$ matrix. Any nontrivial vector that solves $Ax = \lambda x$ is called an **eigenvector**. If $Ax = \lambda x$ has a non-trivial solution, λ is an **eigenvalue**.

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The Eigenvalues of a matrix are the solutions to $det(A - \lambda I_n) = 0$, the**characteristic equation**.

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For each eigenvalue λ, solve the linear system $(A - \lambda I_n)x = 0$ to find the eigenvectors.

Diagonalizability

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An $n \times n$ matrix A is **diagonalizable** if there exists an $n \times n$ invertible matrix P and an $n \times n$ diagonal matrix D such that $P^{-1}AP = D$.

When A is diagnolizable, the columns of P are the eigenvectors of A and the diagonal entries of D are the corresponding eigenvalues.

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Example: Verify that the following matrix is diagonalizable.

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

Diagonalizability Theorems

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A $n \times n$ matrix is diagonalizable if and only if it has n linearly independent eigenvectors.

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Note: Not all diagonalizable matrices have n distinct eigenvalues.