Math 240: Eigenvalues and Eigenvectors

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Outline

Today's Goals

1. Know how to interpret matrices as maps from $\mathbb{R}^n$ to $\mathbb{R}^m$.
2. Know how to find eigenvalues.
3. Know how to find eigenvectors.

Linear Maps and Eigenvectors

Key idea from last time: Every matrix is a linear map and every linear map is a matrix.

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Definition

Let λ be a scalar, x be a $n \times 1$ column vector and A be a $n \times n$ matrix. Any nontrivial vector that solves $Ax = \lambda x$ is called an **eigenvector**. If $Ax = \lambda x$ has a non-trivial solution, λ is an **eigenvalue**.

Types of Linear Maps

The following are types of linear maps

1. Reflection about a line in R^2
2. Reflection about a plane in R^3
3. Orthogonal projection onto an axis in R^2
4. Orthogonal projection onto a plane in R^3
5. Rotation about the origin in R^2
6. Rotation about a line in R^3

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Key idea: Eigenvectors are vectors sent to scalar copies of themselves under the linear map corresponding to A.

How to find Eigenvalues

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Hence, to find the eigenvalues, we solve the polynomial equation $det(A - \lambda I_n) = 0$ called the **characteristic equation**.

Finding Eigenvectors

For each eigenvalue λ, solve the linear system $(A - \lambda I_n)x = 0$ to find the eigenvectors.