Math 240: Systems of Linear Equations and Row-Echelon Form

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Outline

1 Systems of Linear Equations

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Linear systems are essential to finding quantatative or approximate solutions to any problem that can be stated mathematically

Solutions to linear systems

Not every linear system has a unique solution.

Definition

A linear system is called **consistent** if it has a solution, it is called **inconsistent** if it does not have a solution.

There are three possibilities:

1. The system has ∞-many solutions.
2. The system has a unique solution.
3. The system has no solution.

Solving a linear system

The standard way is to use elementary operations to isolate each variable.

The elementary operations are:

1. Multiply an equation by a non-zero constant.
2. Add a non-zero multiple of one equation to another.

Echelon Forms

Definition

A matrix is in **row-echelon form** if

1. Any row consisting of all zeros is at the bottom of the matrix.
2. For all non-zero rows the leading entry must be a one. This is called the **pivot**.
3. In consecutive rows the pivot in the lower row appears to the right of the pivot in the higher row.

Definition

A matrix is in **reduced row-echelon form** if it is in row-echelon form and every pivot is the only non-zero entry in its column.

Row Operations

We will be applying row operations to augmented matrices to find solutions to linear equations. This is called **Gaussian** or **Gauss-Jordan** elimination.

Here are the row operations:

1. Multiply a row by a number.
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Key Fact: If you alter an augmented matrix by row operations you preserve the set of solutions to the linear system.