Math 240: More of Green's Theorem and Surface Integrals

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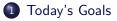
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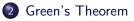
Friday January 27, 2012

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Outline









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Today's Goals

- Investigate the implications of Green's Theorem
- Be able to calculate surface area. 2
- Be able to calculate surface integrals.

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Theorem (Green's Theorem)

Suppose C is a piecewise smooth simple closed curve bounding a region R. If P, Q, $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ are continuous on R, then

$$\oint_{\mathcal{C}} \mathcal{P} dx + \mathcal{Q} dy = \int \int_{\mathcal{R}} (\frac{\partial \mathcal{Q}}{\partial x} - \frac{\partial \mathcal{P}}{\partial y}) dA,$$

where C is oriented counterclockwise.

Green's Theorem: Even when it fails it wins.

Example: Evaluate the following integral where *C* is the positively oriented ellipse $x^2 + 4y^2 = 4$.

$$\oint_C \frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy$$

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The Symmetry Trick

Example:Evaluate the following integral where C is the positively oriented square with vertices (1, 1), (1, -1), (-1, 1), (-1, -1).

$$\oint_C y e^{x^2} dx + x e^{y^2} dy$$

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Methods of Evaluating a Line Integral

- Parametrization and substitution.
- Find a Primitive.
- Use Green's Theorem directly.
- Use Green's Theorem indirectly to simplify the curve you are integrating along.

Surface Area

Definition

Let f(x, y) be a function with continuous partial derivatives f_x and f_y defined on a region R. The **Area of the surface** z = f(x, y) **over** R is given by

$$\int \int_{R} \sqrt{1 + (f_{x}(x, y))^{2} + (f_{y}(x, y))^{2}} dA.$$

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Example: Calculate the surface area of the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane.

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Just like line integral generalizes arc length integral, surface integral generalizes surface area integral.

Definition

Let G be a scalar function and S be a surface given by the graph of z = f(x, y) over the region R. The **surface integral of** G **over** S is given by:

$$\int \int_{S} G(x, y, z) ds =$$

$$\int \int_{R} G(x, y, f(x, y)) \sqrt{1 + (f_x(x, y))^2 + (f_y(x, y))^2} dA$$

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