Math 240: More of Green's Theorem

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Outline

Today's Goals

1. Investigate the implications of Green's Theorem

Green's Theorem

Theorem (Green's Theorem)

Suppose C is a piecewise smooth simple closed curve bounding a region R. If P, Q, $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ are continuous on R, then

$$\oint_C P dx + Q dy = \int \int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA,$$

where C is oriented counterclockwise.

Making Impossible Line Integrals Possible

Example: Evaluate the following line integral on the triangle with vertices $(-1, 1)$, $(0, 1)$ and $(0, 0)$.

$$\oint_C e^{x^2} dx + 2tan^{-1}(x)dy$$

Boundary and Orientation

Definition

Given a region R in $\mathbb{R}^n$, the **boundary** of R, denoted ∂R, is the collection of all points that are adjacent to both R and the complement of R.

Definition

Given a region R in the plane ∂R is a collection of curves. The **positive orientation** on ∂R is a choice of direction on each curve so that the region is always on your left as you move along any curve in the given direction.

Green's theorem holds for regions with multiple boundary curves

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Example: Let C be the positively oriented boundary of the annular region between the circle of radius 1 and the circle of radius 2. Evaluate

$$\oint_C (4x^2 - y^3)dx + (x^3 + y^3)dy$$

Green's Theorem: Even when it fails it wins.

Example: Evaluate the following integral where C is the positively oriented ellipse $x^2 + 4y^2 = 4$.

$$\oint_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$$