# Math 240: Double Integrals in Polar Coordinates and Green's Theorem

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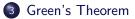
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Review the calculation of double integrals in polar coordinates.Review Green's Theorem.

Image: A matrix and a matrix

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### **Review Problem**

**Example** For the region *R* bounded by y = x, x + y = 4 and x = 0evaluate

$$\int \int_R x + 1 dA$$

## Evaluation of Double Integrals in Polar Coordinates

#### Theorem

Let f be continuous on a region R. If R is Type PI, then

$$\int \int_{R} f(r,\theta) dA = \int_{\alpha}^{\beta} \int_{g_{1}(\theta)}^{g_{2}(\theta)} f(r,\theta) r dr d\theta$$

If R is Type PII, then

$$\int \int_{R} f(r,\theta) dA = \int_{a}^{b} \int_{h_{1}(r)}^{h_{2}(r)} f(r,\theta) r d\theta dr$$

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## Change of Coordinates

If a region in the plane can be describe in polar coordinates as

$$0 \leq g_1(\theta) \leq r \leq g_2(\theta), \ lpha \leq eta \leq eta$$

then we have the following conversion formula

$$\int \int_{R} f(x, y) dA = \int_{\alpha}^{\beta} \int_{g_{1}(\theta)}^{g_{2}(\theta)} f(r\cos(\theta), r\sin(\theta)) r dr d\theta$$

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Example Evaluate

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$$

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#### Theorem (Green's Theorem)

Suppose C is a piecewise smooth simple closed curve bounding a region R. If P, Q,  $\frac{\partial P}{\partial y}$  and  $\frac{\partial Q}{\partial x}$  are continuous on R, then

$$\oint_{\mathcal{C}} \mathcal{P} dx + \mathcal{Q} dy = \int \int_{\mathcal{R}} (\frac{\partial \mathcal{Q}}{\partial x} - \frac{\partial \mathcal{P}}{\partial y}) dA,$$

where C is oriented counterclockwise.