Math 240: Syllabus and Vector Functions

Ryan Blair

University of Pennsylvania

Wednesday January 11, 2012

Outline

Welcome

Adding the Course

Speak to Robin Toney in the Math office on the 4th floor of DRL.

Space is limited.

Syllabus Highlights

1. My contact info
2. TA's contact info
3. Three lectures and 1 recitation a week

Classroom Decorum:(Common Courtesy)

1. No Talking
2. No Texting
3. Cellphone Ringers Off
4. Laptops only used for taking notes

Classroom Decorum:(Common Courtesy)

1. No Talking
2. No Texting
3. Cellphone Ringers Off
4. Laptops only used for taking notes

If the need to do any of the above becomes too great please step outside

Course Webpage

http://www.math.upenn.edu/∼ryblair/Math240S12/index.html

Here you will find

1. Lecture slides
2. Homework assignments
3. A copy of the syllabus
4. A link to Blackboard (were your quiz homework and test scores are posted)
5. Other useful links

Email

1. Include Math 240 in the subject line
2. Send it from a Penn account
3. The body should include your name and your recitation number
4. Allow 24 hrs for a reply
5. Direct homework and quiz questions to your TA, everything else to me

Text

Advanced Engineering Mathematics, 3rd Ed. Dennis Zill and Michael Cullen,

ISBN-13: 978-0-7637-4591-2

No bundle necessary.

Grading

1. 10% Homework
2. 10% Quizzes
3. 20% Midterm 1
4. 25% Midterm 2
5. 35% Final

Homework

1. Homework will be assigned each Monday based on that week's lectures.
2. You can find the current homework assignment on the course website.
3. Homework will be collected each Friday's lecture 11 days after it is assigned.
4. Half the homework score is based on completeness and half on correctness.

Quiz

1. There will be a quiz in each recitation.
2. Anything covered in the previous week is fair game for that weeks quiz.

Quiz

1. There will be a quiz in each recitation.
2. Anything covered in the previous week is fair game for that weeks quiz.
3. **Next weeks quiz question will be based on the material found in the syllabus.**

Exams

Mark your calendars

1. Midterm 1: Feb. 10
2. Midterm 2: Mar. 23
3. Final: May 4

Vector Valued Functions

Goals

1. Review vector valued functions.
2. Introduce del and grad.

Parametric Curves in the Plane

Definition

A parametric curve in the plane is defined by a pair of continuous functions $x = f(t)$ and $y = g(t)$ together with a range for t. (i.e. $a \leq t \leq b$)

Parametric Curves in the Plane

Definition

A parametric curve in the plane is defined by a pair of continuous functions $x = f(t)$ and $y = g(t)$ together with a range for t. (i.e. $a \leq t \leq b$)

Example: What is a parametrization of the circle in the plane?

Parametric Curves in the Plane

Definition

A parametric curve in the plane is defined by a pair of continuous functions $x = f(t)$ and $y = g(t)$ together with a range for t. (i.e. $a \leq t \leq b$)

Example: What is a parametrization of the circle in the plane?

1. The graph of y=f(x) is a parametric curve.
2. A parametric curve is not determined by its graph

Parametric Curves in the Plane

Definition

A parametric curve in the plane is defined by a pair of continuous functions $x = f(t)$ and $y = g(t)$ together with a range for t. (i.e. $a \leq t \leq b$)

Example: What is a parametrization of the circle in the plane?

1. The graph of y=f(x) is a parametric curve.
2. A parametric curve is not determined by its graph

We can combine the data that defines a parametric curve into a *vector-valued* function as

$$r(t) =< f(t), g(t) > \quad a \leq t \leq b$$

Vector-Valued Functions

Definition

Vectors whose components are functions of a parameter t are called **vector-valued** functions.

$$r(t) =< f(t), g(t) >= f(t)\mathbf{i} + g(t)\mathbf{j}$$

$$r(t) =< f(t), g(t), h(t) >= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{i}$$

Vector-Valued Functions

Definition

Vectors whose components are functions of a parameter t are called **vector-valued** functions.

$$r(t) =< f(t), g(t) >= f(t)\mathbf{i} + g(t)\mathbf{j}$$

$$r(t) =< f(t), g(t), h(t) >= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{i}$$

Example: $r(t) =< cos(t), sin(t), t >$

Vector-Valued Functions

Definition

Vectors whose components are functions of a parameter t are called **vector-valued** functions.

$$r(t) =< f(t), g(t) >= f(t)\mathbf{i} + g(t)\mathbf{j}$$

$$r(t) =< f(t), g(t), h(t) >= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{i}$$

Example: $r(t) =< cos(t), sin(t), t >$
Important: These are the parameterized curves we will integrate along

Derivative of a Vector-Valued Function

Definition

If $r(t) =< f(t), g(t), h(t) >$ where f, g, and h are differentiable, then

$$r'(t) =< f'(t), g'(t), h'(t) >$$

Derivative of a Vector-Valued Function

Definition

If $r(t) =< f(t), g(t), h(t) >$ where f, g, and h are differentiable, then

$$r'(t) =< f'(t), g'(t), h'(t) >$$

$r'(t)$ is the **velocity** of $r(t)$ and points in the direction of motion.

Derivative of a Vector-Valued Function

Definition

If $r(t) =< f(t), g(t), h(t) >$ where f, g, and h are differentiable, then

$$r'(t) =< f'(t), g'(t), h'(t) >$$

$r'(t)$ is the **velocity** of $r(t)$ and points in the direction of motion.

Theorem

(Chain Rule) If $\mathbf{r}$ *is a differentiable vector function and* $s = u(t)$ *is a differentiable scalar function, then*

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{s}}{ds}\frac{ds}{dt} = \mathbf{r}'(s)u'(t).$$

Derivative of a Vector-Valued Function

Definition

If $r(t) =< f(t), g(t), h(t) >$ where f, g, and h are differentiable, then

$$r'(t) =< f'(t), g'(t), h'(t) >$$

$r'(t)$ is the **velocity** of $r(t)$ and points in the direction of motion.

Theorem

(Chain Rule) If $\mathbf{r}$ *is a differentiable vector function and* $s = u(t)$ *is a differentiable scalar function, then*

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{s}}{ds}\frac{ds}{dt} = \mathbf{r}'(s)u'(t).$$

Example Find the velocity of $< cos(tan(t)), sin(tan(t)), tan(t) >$.

Partial derivatives

Definition

Given a function $w = f(x, y, z)$, the partial derivative w.r.t. x is

$$\frac{\partial w}{\partial x} = lim_{\Delta x \to 0} \frac{f(x + \Delta x, y, z) - f(x, y, z)}{\Delta x}$$

Partial derivatives

Definition

Given a function $w = f(x, y, z)$, the partial derivative w.r.t. x is

$$\frac{\partial w}{\partial x} = lim_{\Delta x \to 0} \frac{f(x + \Delta x, y, z) - f(x, y, z)}{\Delta x}$$

In practise, to find $\frac{\partial w}{\partial x}$ we differentiate $f(x, y, z)$ with respect to x and assume y and z represent constants.

Partial derivatives

Definition

Given a function $w = f(x, y, z)$, the partial derivative w.r.t. x is

$$\frac{\partial w}{\partial x} = lim_{\Delta x \to 0} \frac{f(x + \Delta x, y, z) - f(x, y, z)}{\Delta x}$$

In practise, to find $\frac{\partial w}{\partial x}$ we differentiate $f(x, y, z)$ with respect to x and assume y and z represent constants.
In the case $z = f(x, y)$, $\frac{\partial z}{\partial x}$ is the slope of the curve of intersection between $z = f(x, y)$ and $y = c$ where c is some constant.

Del and Grad

The differential operator **del** is given by

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Del and Grad

The differential operator **del** is given by

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Given a scalar function $f(x, y, z)$ we can form the **gradient of f** using del.

$$grad(f) = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

Del and Grad

The differential operator **del** is given by

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Given a scalar function $f(x, y, z)$ we can form the **gradient of f** using del.

$$grad(f) = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

∇f points in the direction of greatest change of f.

Del and Grad

The differential operator **del** is given by

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Given a scalar function $f(x, y, z)$ we can form the **gradient of f** using del.

$$grad(f) = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

∇f points in the direction of greatest change of f.
Example: Guess the gradient of $f(x, y, z) = xyz$ at $(1, 1, 1)$ by interpreting the function as volume of a box.