Math 240 Practise Final Fall 2012

Name (printed):

TA:

Recitation Time:

Please *turn off all electronic devices*. You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. **Show all work**, even on multiple choice or short answer questions—the grading will be based on your work shown as well as the end result. Please **clearly mark** a multiple choice option for each problem. Remember to put your name at the top of this page. Good luck.

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

Your signature

Problem	Score (out of)
1	(10)
2	(10)
3	(10)
4	(10)
5	(10)
6	(10)
7	(10)
8	(10)
9	(10)
10	(10)
Total	(80)

1. (10 pts) Calculate the flux of F across S if $F(x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$ oriented outward.

$A)$ 0
$B)$ $-\frac{\pi}{4}$
$C)$ $\frac{11\pi}{8}$
$D)$ 3π
$E)$ $\frac{9\pi}{4}$

2. (10 pts) Given $F(x,y) = \frac{-y\mathbf{i}+x\mathbf{j}}{(x^2+y^2)}$ and C is the positively oriented square with vertices $(-a,0),(a,0),(0,-a),(0,a)$, find the following

$$lim_{a\to 0} \oint_C F \circ dr$$

$A)$ 0
$B)$ 2π
$B)$ $\frac{1}{2}$
$C)$ $-\pi$
$D)$ ∞
$E)$ DNE

3. (10 pts) Let $f(x)$ be the solution to the following IVP.

$$y'' - 4y' + 4y = e^{2x} \;\; y(0) = 1 \;\; y'(0) = 1$$

What is $f(1)$.
$A)$ 1
$B)$ $\frac{e^2}{2}$
$B)$ $\frac{1}{2}$
$C)$ e^2
$D)$ $\frac{-e^2}{2}$
$E)$ $\frac{-e^2}{4}$

4. (10 pts) Please circle "T" for true or "F" for false in the space provided to the left of the following statements. You **DO NOT** need to justify your answer for full credit.

(T F) Every 2×2 diagonalizable matrix with repeated eigenvalue is a diagonal matrix.

(T F) There is a vector field F such that $curl(F) =< x, y, z >$.

(T F) If $det(A) = 0$, then the system $Ax = 0$ has ∞-many solutions.

(T F) If y_1 and y_2 are solutions to a non-homogeneous linear differential equation, then $y_1 + y_2$ is also a solution.

(T F) If A and B are square matrixes such that $AB^2 = I$, then B is invertible.

5. (10 pts) What are the possible eigenvalues of an $n \times n$ matrix A if A satieties the following matrix equation?

$$A - 2I = -A^2$$

$A)$ $0, 1$
$B)$ $0, 2$
$C)$ $0, 1, -2$
$D)$ $1, -2$
$E)$ $1, -3$

Is A invertible? (Yes No)

6. (10pts) Find the general solution to the system of linear ODE

$$\frac{d}{dt}u(x) = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} u(x), \quad where \quad u(x) = \begin{pmatrix} u_1(x) \\ u_2(x) \\ u_3(x) \end{pmatrix}$$

$u(x) =$ ____

7.(10pts)
(a) Give an example of a 3×3 matrix A which has only two eigenvalues and A is *not* diagonalizable. In other words, there does not exist an invertible 3×3 matrix C such that $C^{-1} \cdot A \cdot C$ is a diagonal matrix. **Justify your answer**.

$A =$ ____________________

(b) Give an example of a 3×3 matrix B which has only two eigenvalues and B is diagonalizable. **Justify your answer**.

$A =$ ____________________

8.(10pts) Find A^{201} if

$$A = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$$

$A^{201} =$ ________________.

9.(10pts)Produce a basis of all skew symmetric 3×3 matrices that includes the following matrix. Show that set you produce is a basis.

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

10.(10pts) Find matrices A, B, C, D with the following properties

1. A is invertible and diagonalizable
2. B is invertible and NOT diagonalizable
3. C is NOT invertible and diagonalizable
4. D is NOT invertible and NOT diagonalizable

Scratch Paper

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