MATH 240 - Fall 2012
Practice Midterm Two

Name:

TA:

Recitation number:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	**Score (out of)**
1	(10)
2	(10)
3	(10)
4	(10)
5	(10)
Total	

1.(10 pt) Please mark "T" for true and "F" for false in the space provided to the left of the following statements. If the statement is false, **YOU MUST PROVIDE A COUNTEREXAMPLE FOR FULL CREDIT**.

_____If A is an $n \times n$ matrix such that all of the eigenvalues of A are purely imaginary, then n is even.

_____The vector space of all $n \times n$ skew-symmetric matrices has dimension $n(n-1)/2$.

_____If zero is an eigenvalue of an $n \times n$ matrix A, then A is invertible.

_____If A is an $n \times n$ matrix, then $rowspace(A) = colspace(A)$.

_____The mapping $T : M_n(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = det(A)$ is a linear transformation.

2. Derive the formula for the determinant of a 3×3 matrix by using the definition of determinant.

3. Find all of the eigenvalues and eigenvectors of the following matrix:

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

4.Using the standard definitions of function addition and scalar times function, show that the set of all linear transformations from a vector space V to a vector space W is itself a vector subspace of the vector space of all continuous mappings from V to W.

5. Find a basis for the subspace of $M_2(\mathbb{R})$ consisting of all 2×2 matrices of trace zero. Prove that the set you find is a basis.