

MATH 240 - Fall 2012

Practice Midterm One

Name:

TA:

Recitation number:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	Score (out of)
1	(10)
2	(10)
3	(10)
4	(10)
5	(10)
Total	

1.

A) If \mathbf{F} is a C^1 vector field, show that $\operatorname{div}(\operatorname{curl}(\mathbf{F})) = 0$. Make sure to carefully justify your answer.

For parts B) and C), let S be a closed surface bounding a solid region R and \mathbf{F} be a C^1 vector field.

B) Use Stokes's Theorem to find $\int \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$. **Carefully justify your answer.**

C) Use Gauss's Theorem to find $\int \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$. **Carefully justify your answer.**

2. Given a $m \times n$ matrix A and a $n \times p$ matrix B , show that $(AB)^T = B^T A^T$.

3. Find the inverse of the following matrix or justify that no inverse exists.

$$\begin{pmatrix} 1 & -1 & 2 & 3 \\ 2 & 0 & 3 & -4 \\ 3 & -1 & 7 & 8 \\ 3 & -1 & 5 & -1 \end{pmatrix}$$

4.

a) Given a surface S in 3-space, what property of S does $\iint_S dS$ measure?

b) Let $\mathbf{F} = \langle yz + x, xz + 2x, xy + 3x \rangle$. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the intersection of the plane $2x + y - 3z = 0$ and the sphere $x^2 + y^2 + z^2 = 4$ oriented positively when viewed from above.

5. Evaluate

$$\oint_C \frac{-y}{x^2 + (y-1)^2} dx + \frac{x}{x^2 + (y-1)^2} dy$$

where C is the positively oriented circle of radius 2 centered at the origin.