Math 240: Syllabus and Measuring Vector Fields

Ryan Blair

University of Pennsylvania

Wednesday September 5, 2012

Outline

Welcome

Adding the Course

Speak to Robin Toney in the Math office on the 4th floor of DRL.

Space is limited.

Syllabus Highlights

1. My contact info
2. TA's contact info
3. Three lectures and 1 recitation a week

Classroom Decorum:(Common Courtesy)

1. No Talking
2. No Texting
3. Cellphone Ringers Off
4. Laptops only used for taking notes

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If these constraints are too much, feel free to step outside.

Course Webpage

http://www.math.upenn.edu/∼ryblair/Math240S12/index.html

Here you will find

1. Lecture slides
2. Homework assignments
3. A copy of the syllabus
4. A link to Blackboard (were your quiz homework and test scores are posted)
5. Other useful links

Email

1. Include Math 240 in the subject line
2. Send it from a Penn account
3. The body should include your name and your recitation number
4. Allow 24 hrs for a reply
5. Direct quiz questions to your TA, everything else to me

Text

Required Text: Differential Equations and Linear Algebra, 3rd Ed., Stephen W. Goode and Scott A. Annin,

ISBN-13: 978-0130457943.

Secondary text available on Blackboard: Vector Calculus, 4th Ed., by Susan Jane Colley

Grading

1. 0% Homework
2. 20% Quizzes
3. 20% Midterm 1
4. 25% Midterm 2
5. 35% Final

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Course grades are curved using the final exam in accordance with the math departments 30-30-30-10 policy.

Homework

1. Homework will be assigned each week based on the material covered that week.
2. You can find the current homework assignment on the course website.
3. Homework will not be collected or graded.

Quiz

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2. Quiz questions will be, possibly slight variations on, homework problems assigned the previous week.

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3. **Next week's quiz question will be based on the material found in the syllabus.**

Exams

Mark your calendars

1. Midterm 1: Oct. 8
2. Midterm 2: Nov. 12
3. Final: Dec. 18

Measuring Vector Fields

Definition

A 3-dimensional vector field is a map from $\mathbb{R}^3$ to $\mathbb{R}^3$ denoted by

$$F(x,y,z) =< f(x,y,z), g(x,y,z), h(x,y,z) >$$

where $f(x,y,z)$, $g(x,y,z)$ and $h(x,y,z)$ and scalar valued functions.

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Vector fields appear everywhere from Magnetic fields to fluid flows

Del and Grad

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Given a scalar function $f(x, y, z)$ we can form the **gradient of f** using del.

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Example: Guess the gradient of $f(x,y,z) = xyz$ at $(1,1,1)$ by interpreting the function as volume of a box.

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Divergence measures the tendency of a vector field to expand or contract.

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The **curl** of a vector field $F = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ is the **vector field**

$$curl(F) = \nabla \times F = (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\mathbf{i} + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\mathbf{j} + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\mathbf{k}$$

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The magnitude of Curl is the intensity of the rotation about a point. The direction of Curl is the axis of maximal rotation about a point.

Curl in Dimension 2

Question: What is curl in dimension 2?

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Theorem (Green's Theorem)

Suppose C is a piecewise smooth simple closed curve bounding a region R. If P, Q, $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ are continuous on R, then

$$\oint_C Pdx + Qdy = \int\int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA,$$

where C is oriented counterclockwise.

Domain and Range

It is important to note

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Homework: If F is a 3-dimensional vector field show

$$div(curl(F)) = 0$$