Math 240: Matrix Basics

Ryan Blair

University of Pennsylvania

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Outline

1. Operations on Matrices

Operations on Matrices

Goals

1. Matrix basics
2. Add and subtract matrices
3. Multiply a matrix by a scalar
4. Multiply matrices
5. Take the transpose of a matrix
6. Special types of matrices
7. Matrix properties

A Quick Review

Definition

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The **dimension** of a matrix is (the number of Rows)×(the number of columns). Two Matrices are equal if they have the same dimension and corresponding entries are equal.

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3. Matrix multiplication: The ij entry is the dot product of the i-th row of the matrix on the left with the j-th column of the matrix on the right.
4. Matrix Transpose: $(a_{ij})^T_{m\times n} = (a_{ji})_{n\times m}$ (Rows of A become columns of A^T and columns of A become rows of A^T.)

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The **identity matrix of dimension** n, denoted I_n, is the $n \times n$ diagonal matrix where all the diagonal entries are 1.

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Definition

The **trace** of a square matrix is the sum of the diagonal entries.

Matrix Properties

Let A and B be $m \times n$ matrices. Let k and p be scalars.

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $k(A + B) = kA + kB$
4. $(k + p)A = kA + pA$

Let 0 be the $m \times n$ matrix of all zeros

1. $A + 0 = A$
2. $A - A = 0$
3. $kA = 0$ implies $k = 0$ or $A = 0$.

More Matrix Properties

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AC + BC$
4. $k(AB) = (kA)B = A(kB)$
5. $I_m A = A$
6. $A I_n = A$

Even More Matrix Properties

1. $(A^T)^T = A$
2. $(kA)^T = kA^T$
3. $(A + B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T$