# Math 240: Stokes's Theorem and Gauss's Theorem Again

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Monday Sept. 26, 2012

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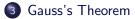
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## Today's Goals

# Be able to apply Stokes's Theorem

### Be able to apply Gauss's Theorem

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## Stokes' Theorem

#### Theorem

Let S be a **nice** oriented surface in  $\mathbb{R}^3$ . Suppose that  $\partial S$  is oriented consistently with S. Let **F** be a vector field of class  $C^1$  defined on S. Then

$$\int \int_{\mathcal{S}} (
abla imes \mathbf{F}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

**Example:** Let  $F = \langle y^2, 2z + x, 2y^2 \rangle$ . Find a plane ax + by + cz = 0 such that  $\oint_C F \circ dr = 0$  for every smooth simple closed curve *C* in the plane.

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## Gauss's Theorem

#### Theorem

Let D be a bounded solid region in  $\mathbb{R}^3$  whose boundary  $\partial D$  consists of finitely many piecewise smooth, closed orientable surfaces, each of which is oriented by unit normals that point away from D. Let **F** be a vector field of class  $\mathbb{C}^1$  whose domain includes D. Then

$$\int \int_{\partial D} \mathbf{F} \cdot d\mathbf{S} = \int \int \int_{D} \nabla \cdot \mathbf{F} dV.$$

**Example** Find the flux of  $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$  outward through the surface of the cube cut from the first octant by the planes x = 1, y = 1 and z = 1.

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## Gauss's Theorem

#### Theorem

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**Example:** Use the Gauss's theorem to evaluate  $\int \int_{S} \mathbf{F} \cdot d\mathbf{S}$  where  $\mathbf{F} = \langle x + y, z, z - x \rangle$  and *S* is the boundary of the region between  $z = 9 - x^2 - y^2$  and the *xy*-plane.

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## Gauss's Theorem

#### Theorem

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$$\int \int_{\partial D} \mathbf{F} \cdot d\mathbf{S} = \int \int \int_{D} \nabla \cdot \mathbf{F} dV.$$

Example Find the outward flux of

$$\frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

across the region D given by  $0 < a^2 \le x^2 + y^2 + z^2 \le b^2$ .

#### Theorem

Let S be a bounded, piecewise smooth, oriented surface in  $\mathbb{R}^3$ . Suppose that  $\partial S$  consists of finitely many piecewise  $C^1$ , simple, closed curves each of which is oriented consistently with S. Let **F** be a vector field of class  $C^1$  defined on S. Then

$$\int \int_{\mathcal{S}} (
abla imes {f F}) \cdot d{f S} = \oint_{\partial \mathcal{S}} {f F} \cdot d{f s}$$

#### Theorem

Let D be a bounded solid region in  $\mathbb{R}^3$  whose boundary  $\partial D$  consists of finitely many piecewise smooth, closed orientable surfaces, each of which is oriented by unit normals that point away from D. Let **F** be a vector field of class  $\mathbb{C}^1$  whose domain includes D. Then

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