Math 240: Undetermined Coefficients

Ryan Blair

University of Pennsylvania

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Outline

Today's Goals

1. Use the method of undetermined coefficients to solve nonhomogeneous Constant Coefficient Linear Differential Equations.

The Method of Undetermined Coefficients

Given a constant coefficient nonhomogeneous differential equation

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Given a constant coefficient nonhomogeneous differential equation

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2. Step 2: Find a particular solution by analyzing g(x) and making an educated guess.
3. Step 3: Add the homogeneous solution and the particular solution together to get the general solution.

The form of y_p is a linear combination of all linearly independent functions that are generated by repeated differentiation of $g(x)$.

Guessing Particular Solutions

g(x)	**Guess**
constant	A
$3x^2 - 2$	$Ax^2 + Bx + C$
Polynomial of degree n	$A_n x^n + A_{n-1} x^{n-1} + ... + A_0$
$cos(4x)$	$Acos(4x) + Bsin(4x)$
$Acos(nx) + Bsin(nx)$	$Acos(nx) + Bsin(nx)$
e^{4x}	Ae^{4x}
$x^2 e^{5x}$	$(Ax^2 + Bx + C)e^{5x}$
$e^{2x} cos(4x)$	$Ae^{2x} sin(4x) + Be^{2x} cos(4x)$
$3xsin(5x)$	$(Ax + B)sin(5x) + (Cx + D)cos(5x)$
$xe^{2x} cos(3x)$	$(Ax + B)e^{2x} sin(3x) + (Cx + D)e^{2x} cos(3x)$

A Problem

Solve $y'' - 5y' + 4y = 8e^x$ using undetermined coefficients.

The solution

When the natural guess for a particular solution duplicates a homogeneous solution, multiply the guess by x^n, where n is the smallest positive integer that eliminates the duplication.