Math 240: Systems of Differential Equations, Repeated Eigenvalues

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Wednesday November 21, 2012

Outline

Today's Goals

1. Solve linear systems of differential equations with non-diagonalizable coefficient matrices.

Repeated Eigenvalues

In a $n \times n$, constant-coefficient, linear system there are two possibilities for an eigenvalue λ of multiplicity 2.

1. λ has two linearly independent eigenvectors $\mathbf{K}_1$ and $\mathbf{K}_2$.
2. λ has a single eigenvector $\mathbf{K}$ associated to it.

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In the second case, there are linearly independent solutions $\mathbf{K} e^{\lambda t}$ and

$$[\mathbf{K} t e^{\lambda t} + \mathbf{P} e^{\lambda t}]$$

where we find $\mathbf{P}$ be solving $(\mathbf{A} - \lambda \mathbf{I})\mathbf{P} = \mathbf{K}$. $\mathbf{P}$ is a **generalized eigenvector**.

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where we find **P** be solving $(\mathbf{A} - \lambda\mathbf{I})\mathbf{P} = \mathbf{K}$
Exercise: Solve the linear system $X' = AX$ if

$$A = \begin{pmatrix} -8 & -1 \\ 16 & 0 \end{pmatrix}$$

How Bad Can it Get?

In general, you will only be asked to solve systems $X' = AX$ if the multiplicity of the eigenvalues of A is at most 1 more than the number of linearly independent eigenvectors for that value. In this case you need to find at most one vector $\mathbf{P}$ such that $(\mathbf{A} - \lambda\mathbf{I})\mathbf{P} = \mathbf{K}$

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Exercise: Solve the linear system $X' = AX$ if

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$