# Math 240: Linear Transformations of $\mathbb{R}^2$

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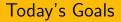


### 2 Linear Transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ .

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### • Know linear transformations of $\mathbb{R}^2$ .

# Matrices ARE linear transformations

### Definition

A mapping  $T: V \rightarrow W$  is a **linear transformation** if the following hold:

$$T(u+v) = T(u) + T(v) \text{ for all } u, v \in V$$

2 T(cv) = cT(v) for all  $v \in V$  and all scalars c.

#### Theorem

Let  $T : \mathbb{R}^n \to \mathbb{R}^m$  be a linear transformation. T is completely described by

$$T(v) = Av$$

where A is the  $m \times n$  matrix

$$A = [T(\mathbf{e}_1), T(\mathbf{e}_2), ..., T(\mathbf{e}_n)]$$

and  $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n$  are the standard basis vectors in  $\mathbb{R}^n$ .

Linear Transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ .

# Linear Transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ .

### Reflections

$$\left(\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array}\right), \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right), \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$$

### Stretching

$$\left(\begin{array}{cc}k&0\\0&1\end{array}\right),\left(\begin{array}{cc}1&0\\0&k\end{array}\right)$$

### Shearing

$$\left(\begin{array}{cc}1&k\\0&1\end{array}\right), \left(\begin{array}{cc}1&0\\k&1\end{array}\right)$$

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# How to build any invertible linear transformation

### Definition

If T and R are linear transformations from  $\mathbb{R}^n$  to  $\mathbb{R}^n$  with matrices A and B respectively, then their composition  $T \circ R$  is a linear transforation with matrix AB.

### Theorem

Any linear transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  with invertible matrix is obtained by composing reflections, stretches and shears.